

High-precision Standstill Flux Linkage Identification for Synchronous Reluctance Motors Based on Gradient Projection and Implicit Inductance Modeling

Cheng Ye, *Student Member, IEEE*, Yankai Jia, Yusheng Song, Changxing Guo, and Chuanwen Shen

Abstract—Accurate standstill flux-linkage identification of synchronous reluctance motors (SynRMs) is challenged by voltage-integration errors and optimization interference between data fitting and physical constraints. This paper develops a compact physics-informed neural network (PINN), whose implicit secant-inductance architecture enforces the zero-flux condition and dq-axis parity symmetries by construction. Its training objective combines flux-fitting mean-square error, Maxwell reciprocity, and two axis-wise discrete voltage-equation residuals. Pairwise projecting conflicting gradient (PCGrad) removes conflicting gradient components among these four weighted objectives before parameter update. Following current-injection prealignment, self- and cross-saturation data are acquired by standstill hysteresis voltage injection without mechanical rotor locking or continuous position feedback. The dynamometer-derived constant-speed flux map is excluded from gradient training and serves only as a common external benchmark. Experiments on a 0.55 kW SynRM yield a three-seed average benchmark mean absolute error (MAE) of 0.01248 ± 0.00254 Wb, the lowest among the evaluated neural and function-fitting models. For the complete multi-physics objective, PCGrad reduces the average benchmark MAE by 17.4% relative to direct gradient summation. Ablation studies verify the complementary contributions of reciprocity, the discrete voltage equation, and gradient projection. The resulting direct current-to-flux mapping provides a compact and physically consistent magnetic model for model-based motor-control applications. The implementation code and datasets are available at [\[https://github.com/KrisCCeng/SynRM-PINN.git\]](https://github.com/KrisCCeng/SynRM-PINN.git).

Index Terms—Synchronous reluctance motor (SynRM), Physics-informed neural network (PINN), Flux linkage identification, Projecting conflicting gradient (PCGrad), Standstill test, Physical constraints.

I. INTRODUCTION

ELECTRIC motors serve as the core power sources in modern industry and transportation, and their operating efficiency is directly linked to the realization of global energy conservation and emission reduction goals. Among various motor topologies, the synchronous reluctance motor (SynRM) features a rotor devoid of permanent magnets, offering advantages such as a robust structure, low manufacturing cost, and zero dependence on rare-earth materials [1]–[3]. Consequently, in applications highly sensitive to cost and environmental sustainability, the SynRM has garnered significant attention in recent years as a highly competitive alternative to the permanent magnet synchronous motor (PMSM) [4].

The fundamental prerequisite for achieving high-performance control of SynRMs—such as maximum torque per ampere (MTPA) and sensorless control [5]–[9]—is the establishment of a high-precision magnetic characteristic model [10]. Due to the highly anisotropic nature of the rotor, SynRMs exhibit pronounced magnetic self-saturation and cross-saturation effects. Consequently, the flux linkage is no longer a simple linear function of the current, but rather a highly complex, bivariate nonlinear function. Inaccuracies in the flux linkage model directly lead to deviations in the control operating point, severely degrading overall motor performance [11].

To address the aforementioned nonlinear flux linkage modeling challenges, existing research has primarily evolved along two dimensions: data acquisition and modeling algorithms. Regarding data acquisition, physical model-based methods, such as finite element analysis (FEA), can compute comprehensive flux linkage maps based on the motor's geometric parameters and material properties [12]. However, FEA involves cumbersome modeling and computation processes and requires detailed dimensional and material parameters of the manufactured motor, thereby limiting its generality and engineering practicality [13].

In contrast, experimental data-driven methods offer greater practicality. Several studies [14]–[17] utilize dynamic operating data acquisition to obtain real-time data; however, these methods intrinsically rely on position encoders or dynamometer testbenches. Notably, the constant-speed dynamometer-based flux linkage identification method

Manuscript received May 9, 2026; revised August 13, 2026; accepted August 26, 2026. Date of publication September 25, 2026; Date of current version September 22, 2026.

Cheng Ye, Yankai Jia, Yusheng Song, Changxing Guo, and Chuanwen Shen are with the School of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, China (e-mail: kris.ccheng@foxmail.com; 2296853382@qq.com; sys3853@163.com; 1396856200@qq.com; scw@mail.xjtu.edu.cn).

(Corresponding Author: Chuanwen Shen)

Digital Object Identifier 10.30941/CESSTEMS.2026.00029

proposed by Armando *et al.* [16] can accurately extract the cross-saturation flux linkage model. Nevertheless, its strict dependence on a position encoder and a testbench imposes stringent testing conditions. Alternatively, [18]-[23] acquire data while the motor is at a standstill. Stumberger *et al.* [18] employed the traditional locked-rotor test, but this approach relies on cumbersome mechanical locking devices. Hinkkanen *et al.* [21] and Woo *et al.* [22] utilized a standstill identification method based on hysteresis voltage injection. This method eliminates the need for mechanical rotor locking or position sensors, featuring simple operation and broad applicability. However, the open-loop integration used to calculate the flux linkage is highly susceptible to drift errors, and the micro-vibrations of the rotor induced by high-frequency voltage pulses can degrade data accuracy.

Regarding modeling algorithms, look-up tables (LUTs) [9] and polynomial fitting [23] are currently the most widely applied. However, for a two-dimensional dq map, LUT storage and dense data-acquisition effort grow quadratically with the number of grid points along each current axis, and accurate interpolation requires sufficiently dense and well-distributed measurements over the target current plane, while polynomial models exhibit limited accuracy when describing strongly nonlinear characteristics. Furthermore, both approaches struggle to satisfy physical constraints, such as the reciprocity condition. To overcome the deficiencies of polynomial models, researchers have proposed various complex magnetic saturation function models. For example, Hinkkanen *et al.* [21] adopted an algebraic magnetic model incorporating the reciprocity condition; however, this method necessitates complex numerical inverse operations during practical application. Woo *et al.* [22] introduced a novel saturation approximation function to ensure physical consistency. Nevertheless, due to the extreme complexity of magnetic characteristics, it is often difficult to fully capture the true underlying physical laws using merely mathematical expressions with a limited number of parameters. These approaches are therefore treated as complementary references with different accuracy, acquisition, storage, and online-evaluation characteristics.

In recent years, artificial neural networks (ANNs) have been introduced into the field of motor modeling, owing to their robust nonlinear fitting capabilities [24]-[25]. Lee *et al.* [24] proposed an ANN-based method to identify the flux linkage saturation model. By leveraging the powerful nonlinear mapping ability of neural networks, this approach effectively overcomes the representational limitations of traditional function fitting. However, conventional ANNs are purely data-driven “black-box” models. In the absence of guidance from physical laws, particularly when dealing with noisy experimental data, they are highly prone to overfitting. Consequently, their outputs may violate physical laws such as energy conservation, resulting in limited generalization capability and physical plausibility.

In summary, existing SynRM flux linkage identification schemes still present compromises among accuracy, physical consistency, and experimental complexity. To address this, this paper proposes a high-precision flux linkage

identification framework based on physics-informed neural networks (PINNs). This article is an extended version of our preliminary work presented in [26]. The main contributions of this paper are summarized as follows:

1) A compact implicit secant-inductance network is developed for the nonlinear current-to-flux mapping. Its multiplicative output reconstruction and absolute-current inputs enforce the zero-flux condition and dq-axis parity symmetries by construction, reducing the hypothesis space without introducing additional penalty terms.

2) Maxwell reciprocity and axis-wise discrete voltage-equation residuals are combined with the data loss as four weighted objectives. Pairwise projecting conflicting gradient (PCGrad) removes conflicting gradient components before the Adam update. Multi-seed sensitivity and ablation studies quantify the independent and combined effects of the physics objectives and gradient projection.

3) A cycle-based standstill identification procedure is established for hysteresis-voltage injection without mechanical rotor locking or continuous position feedback. Cross-saturation voltages are selected from self-saturation secant inductances and target dq-axis scan rates. Controlled error propagation, free-motion simulation, multi-window and injection-voltage comparisons, and paired network-size tests define the method’s acquisition and deployment conditions on the investigated motor.

To promote the application and development of physics-informed machine learning in the field of motor drives, the complete network architecture, the gradient projection optimization algorithm, and the experimental datasets of this study have been open-sourced on GitHub. The project repository is available at: <https://github.com/KrisCCeng/SynRM-PINN.git> [27].

II. MAGNETIC CHARACTERISTIC MODEL AND PHYSICAL CONSTRAINTS OF THE SYNRM

This section establishes the mathematical model and electromagnetic nonlinear characteristics of the SynRM, extracting the critical physical constraints (i.e., initial conditions, reciprocity, and symmetry) that the model must strictly adhere to, serving as physical priors to be embedded into the neural network architecture.

A. dq-axis Voltage Equations

To simplify controller design and achieve the decoupling of alternating current (AC) quantities, the dynamic behaviors of the SynRM are typically described in the dq synchronous rotating reference frame. Neglecting hysteresis losses and eddy current effects, the stator voltage equations of the SynRM can be expressed as:

$$\begin{cases} u_d = R_s i_d + \frac{d\psi_d}{dt} - \omega_e \psi_q \\ u_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_e \psi_d \end{cases} \quad (1)$$

where u_d , u_q and i_d , i_q are the d- and q-axis stator voltages and currents, respectively; R_s is the stator winding resistance; ψ_d

and ψ_q are the d- and q-axis flux linkages, respectively; ω_e is the rotor electrical angular velocity.

B. Magnetic Saturation and Cross-saturation Characteristics

Due to the highly anisotropic rotor structure of the SynRM, its magnetic circuit is highly susceptible to the saturation characteristics of the core material. In high-performance control applications, flux linkages can no longer be regarded as linear mappings of the excitation currents; instead, the following complex nonlinear relationships must be rigorously considered:

$$\begin{cases} \psi_d = f_d(i_d, i_q) \\ \psi_q = f_q(i_d, i_q) \end{cases} \quad (2)$$

where f_d and f_q denote the nonlinear d- and q-axis flux-linkage mappings with respect to the two-axis currents, respectively.

C. Key Physical Constraints

To ensure that the data-driven identification model preserves physical consistency to the greatest extent possible, in addition to the voltage equation (1), the following three fundamental physical constraints are extracted:

1) Initial Conditions

Under the ideal assumption of neglecting remanent flux, no flux linkage should exist in the magnetic circuit when there is no current excitation in the stator windings. That is, the model must pass through the origin:

$$f_d(0,0) = 0, \quad f_q(0,0) = 0 \quad (3)$$

This constraint guarantees the zero-point stability of the identified model in the low-current region.

2) Maxwell's Reciprocity

According to Maxwell's relations, in an ideal lossless magnetic circuit, the stored magnetic energy is uniquely determined by the state variables. This implies that the partial derivatives of the flux linkages with respect to the currents must satisfy the reciprocity condition:

$$\frac{\partial \psi_d(i_d, i_q)}{\partial i_q} = \frac{\partial \psi_q(i_d, i_q)}{\partial i_d} \quad (4)$$

This constraint mathematically enforces that the established model complies with the principle of energy conservation, thereby enhancing the physical fidelity of the torque calculations.

3) Symmetry Constraints

Considering the electromagnetic symmetry of the SynRM rotor structure, the d- and q-axis magnetic flux paths are symmetric with respect to their coordinate axes. Specifically, ψ_d is an odd function with respect to i_d and an even function with respect to i_q ; conversely, ψ_q is an even function with respect to i_d and an odd function with respect to i_q . The mathematical expressions are as follows:

$$\begin{cases} \psi_d(i_d, i_q) = -\psi_d(-i_d, i_q) = \psi_d(i_d, -i_q) \\ \psi_q(i_d, i_q) = \psi_q(-i_d, i_q) = -\psi_q(i_d, -i_q) \end{cases} \quad (5)$$

Incorporating these symmetry constraints not only significantly reduces the parameter space that the neural network needs to fit but also enhances the prediction accuracy of the model during current polarity reversals.

III. ARCHITECTURE DESIGN OF THE PHYSICS-INFORMED NEURAL NETWORK

In physics-informed machine learning, restructuring the network architecture to introduce an "inductive bias" [28] is central to enforcing hard physical constraints. By modifying the mapping architecture, this section enables the model to strictly satisfy the zero-point and symmetry physical priors, completely independent of the training data.

A. Implicit Secant Inductance Modeling

Conventional neural networks typically directly approximate the mapping from current to flux linkage, an approach that struggles to guarantee numerical stability at zero current. To overcome these limitations, this paper proposes a PINN architecture based on implicit secant inductance modeling, as illustrated in Part A of Fig. 1.

Instead of allowing the network to directly output predicted flux linkage values, this architecture outputs the secant inductances $\hat{L}_d$ and $\hat{L}_q$ of the d- and q-axes. These quantities are related by $\hat{\psi} = \hat{L}\mathbf{i}$, where $\hat{\psi} = [\hat{\psi}_d, \hat{\psi}_q]^T$ is the predicted flux-linkage vector, $\mathbf{i} = [i_d, i_q]^T$ is the current vector, and $\hat{L} = \text{diag}(\hat{L}_d, \hat{L}_q)$ is the predicted secant-inductance matrix. This multiplicative relationship inherently guarantees from its mathematical structure that the flux linkages are strictly zero when the currents are zero. This effectively eliminates zero-point drift and automatically satisfies the physical constraint expressed in (3). Furthermore, because the secant inductance reflects the degree of saturation within the magnetic circuit, its variation trend is considerably smoother than that of the flux linkages, thereby significantly reducing the fitting difficulty for the neural network.

To enforce the symmetry constraints expressed in (5), the absolute values of the excitation currents ($|i_d|, |i_q|$) are utilized as the actual inputs to the network. This design not only compels the network to inherently satisfy full-plane electromagnetic symmetry but also significantly reduces the parameter search space, thereby enhancing the model's generalization capability across different current polarities. Internally, the predicted inductances are multiplied by the original signed currents to reconstruct the final flux linkage values:

$$\begin{cases} \hat{\psi}_d = \hat{L}_d(|i_d|, |i_q|) \cdot i_d \\ \hat{\psi}_q = \hat{L}_q(|i_d|, |i_q|) \cdot i_q \end{cases} \quad (6)$$

The absolute-value transformation is not classically differentiable at zero. In the implementation, automatic differentiation assigns the zero subgradient, $\partial|x|/\partial x|_{x=0} = 0$. Consequently, for samples on $i_q = 0$ (or $i_d = 0$), the derivative transmitted through the corresponding orthogonal absolute-value input is zero. This agrees with the zero slope of an even flux dependence on the orthogonal-axis current at its symmetry axis. Meanwhile, the multiplicative forms $\psi_d = i_d L_d$ and $\psi_q = i_q L_q$ preserve the self-axis derivatives at zero current. No smoothing approximation is used in the reported results.

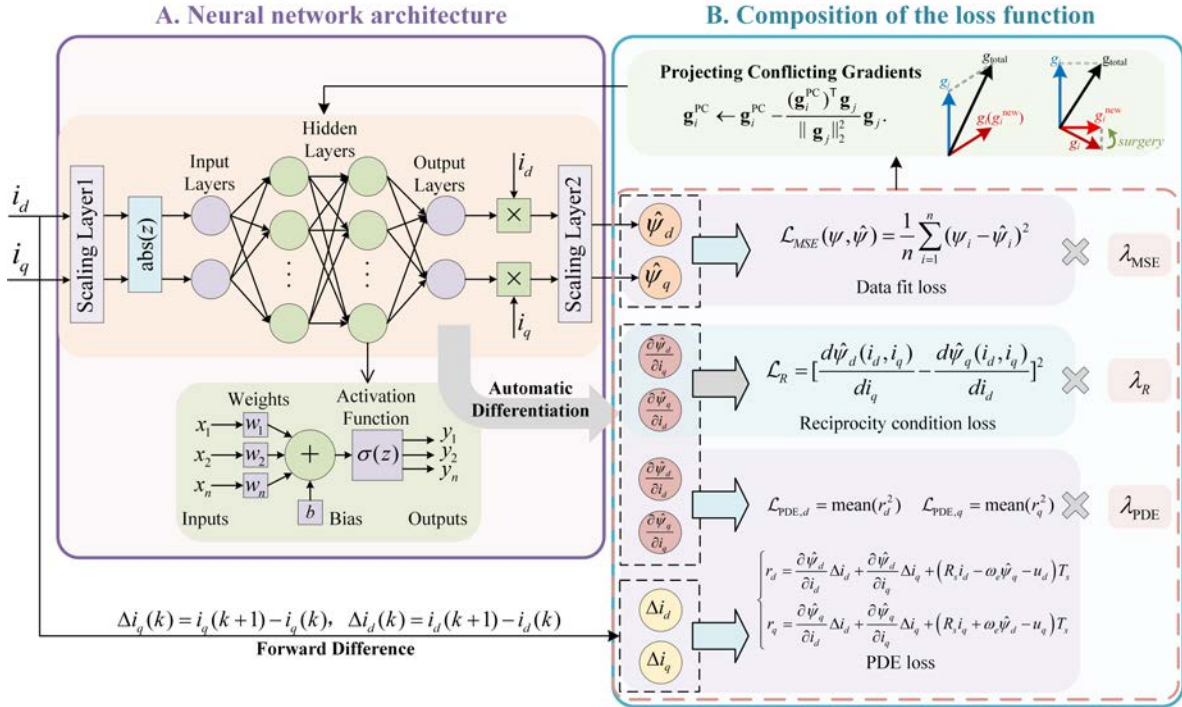


Fig. 1. Proposed implicit secant-inductance network and construction of the four weighted data and physics objectives.

To accommodate the automatic differentiation computations within the PINN framework, the exponential linear unit (ELU) is selected as the activation function for the hidden layers. Its mathematical expression is:

$$f(x) = \begin{cases} x, & x > 0 \\ \alpha(e^x - 1), & x \leq 0 \end{cases} \quad (7)$$

where α is typically set to 1. Compared to the rectified linear unit (ReLU) function, the ELU exhibits superior continuity and smoothness in the negative region, which facilitates the derivative calculations required by the physical loss function. Furthermore, the computational overhead of the ELU is lower than that of complex functions such as Tanh, making it more conducive to efficient model deployment in embedded controllers.

B. Scaling Layer Design

Neural networks are inherently sensitive to the scale of input and output data, and normalization is an effective technique to address this issue. However, directly calculating physical residuals within a dimensionless normalized space inevitably leads to dimensional inconsistency, which fundamentally invalidates the underlying physical laws.

To resolve this conflict, this paper introduces a scaling layer between the neural network and the physical loss function. Let I_{base} and Ψ_{base} denote the base values for the input excitation currents and output flux linkages, respectively (which are selected as the maximum absolute values of the respective inputs and outputs). The base value for the secant inductance is naturally determined as $L_{\text{base}} = \Psi_{\text{base}}/I_{\text{base}}$.

The superscript $*$ is utilized to denote the normalized dimensionless variables. In scaling layer 1, the mapping relationship between the actual input currents and the normalized currents is given by:

$$i_d = i_d^* I_{\text{base}}, \quad i_q = i_q^* I_{\text{base}} \quad (8)$$

Within the dimensionless space, the neural network receives the absolute normalized currents ($|i_d^*|, |i_q^*|$) and outputs the predicted normalized secant inductances $\hat{L}_d^*$ and $\hat{L}_q^*$. Internally, the normalized flux linkages are first computed as $\hat{\psi}_d^* = \hat{L}_d^* i_d^*$ and $\hat{\psi}_q^* = \hat{L}_q^* i_q^*$. Subsequently, scaling layer 2 restores these values to the true flux linkage predictions with correct physical dimensions:

$$\hat{\psi}_d = \hat{\psi}_d^* \Psi_{\text{base}} = (\hat{L}_d^* I_{\text{base}}) \Psi_{\text{base}} \quad (9)$$

As illustrated in Fig. 1, scaling layer 1 ensures that the network trains highly efficiently within the optimal numerical range of $[-1, 1]$. Conversely, scaling layer 2 precisely maps the dimensionless outputs of the neural network back to their true physical dimensions.

The current and flux bases are selected according to the intended identification ranges. An excessively large base compresses the samples into a narrow activation interval, whereas an undersized base places part of the operating range outside the normalized domain. If overload identification is required, the acquisition range, scaling bases, and validation range must be extended together; otherwise, the flux linkage outside the acquired current envelope remains an extrapolation.

IV. MULTI-PHYSICS LOSS AND GRADIENT PROJECTION STRATEGY

In addition to architectural hard constraints, introducing a “learning bias” [28] as a soft constraint within the loss function is another critical mechanism for PINNs to guarantee physical consistency [29]-[30]. In this work, Maxwell reciprocity and the discrete dq-axis voltage equations are introduced as complementary soft objectives. The former

constrains the cross derivatives of the magnetic map, whereas the latter links adjacent measured current-voltage samples to the local network Jacobian.

A. Construction of the Loss Function

The total loss function, $\mathcal{L}_{\text{total}}$, proposed in this study comprises a data fitting loss and multiple physical constraint losses, as illustrated in Part B of Fig. 1. Through the specific design of the network architecture, the initial zero-point constraints and the symmetry constraints of the flux linkages have been inherently satisfied from the underlying mathematical logic. Consequently, it is unnecessary to introduce these two items as additional physical constraint loss terms in the proposed model.

1) Data fitting loss ($\mathcal{L}_{\text{MSE}}$)

This loss term quantifies the discrepancy between the final flux linkage prediction outputs of the neural network and the experimental flux linkage measurements. It is calculated using the mean squared error (MSE):

$$\mathcal{L}_{\text{MSE}} = \frac{1}{2N} \sum_{k=1}^N \left[(\psi_d(k) - \hat{\psi}_d(k))^2 + (\psi_q(k) - \hat{\psi}_q(k))^2 \right] \quad (10)$$

where N is the total number of data points in a training batch; $\psi_x(k)$ denotes the experimental measurement; $\hat{\psi}_x(k)$ represents the predicted value calculated by the network.

2) Reciprocity condition loss ($\mathcal{L}_R$)

The reciprocity constraint serves as the core physical condition to ensure energy conservation within the magnetic circuit. During the neural network training process, automatic differentiation techniques are employed to compute the cross-partial derivatives. Consequently, in accordance with the reciprocity formula in (4), the reciprocity loss is formulated as follows:

$$\mathcal{L}_R = \frac{1}{2N} \sum_{k=1}^N \left(\frac{\partial \hat{\psi}_d(i_d(k), i_q(k))}{\partial i_q} - \frac{\partial \hat{\psi}_q(i_d(k), i_q(k))}{\partial i_d} \right)^2 \quad (11)$$

3) Partial differential equation loss ($\mathcal{L}_{\text{PDE}}$)

For adjacent samples k and $k+1$, the measured current increments are defined as $\Delta i_d = i_d(k+1) - i_d(k)$ and $\Delta i_q = i_q(k+1) - i_q(k)$. The flux increments are approximated from these current increments and the network Jacobian. The axis-specific residuals are:

$$\begin{cases} r_d = \frac{\partial \hat{\psi}_d}{\partial i_d} \Delta i_d + \frac{\partial \hat{\psi}_d}{\partial i_q} \Delta i_q + (R_s i_d - \omega_e \hat{\psi}_q - u_d) T_s \\ r_q = \frac{\partial \hat{\psi}_q}{\partial i_d} \Delta i_d + \frac{\partial \hat{\psi}_q}{\partial i_q} \Delta i_q + (R_s i_q + \omega_e \hat{\psi}_d - u_q) T_s \end{cases} \quad (12)$$

The voltage-equation losses are $\mathcal{L}_{\text{PDE},d} = \text{mean}(r_d^2)$ and $\mathcal{L}_{\text{PDE},q} = \text{mean}(r_q^2)$. The speed-coupling terms vanish at ideal standstill, while the general dq form is retained.

4) Total loss function ($\mathcal{L}_{\text{total}}$)

The four objectives supplied to the multi-task optimizer are $\mathcal{J}_1 = \mathcal{L}_{\text{MSE}}$, $\mathcal{J}_2 = \lambda_{\text{PDE}} \mathcal{L}_{\text{PDE},d}$, $\mathcal{J}_3 = \lambda_{\text{PDE}} \mathcal{L}_{\text{PDE},q}$, $\mathcal{J}_4 = \lambda_R \mathcal{L}_R$ (13)

The coefficients λ_R and λ_{PDE} control the relative contributions of the two physics objectives, and their recommended initial values are determined through the

sensitivity analysis in Section VI-B. The zero-flux and parity constraints are already imposed by the network architecture and therefore do not require additional loss terms.

B. Multi-task Learning Strategy Based on Projecting Conflicting Gradients

In multi-task learning frameworks, such as PINNs, balancing the data fitting loss and the physical constraint loss remains a critical challenge [31]-[32]. The phenomenon of “gradient interference” often emerges in multi-task learning, wherein the optimization process of one task may degrade the accuracy of another, causing the model to struggle with convergence or fall into suboptimal local minima. To address this, this paper introduces the projecting conflicting gradients (PCGrad) method [33], also known as gradient surgery, proposed by Yu *et al.*

Let $g_i = \nabla_{\theta} \mathcal{J}_i$ denote the gradient of the weighted objective $\mathcal{J}_i$, where ∇_{θ} denotes the gradient operator with respect to the trainable network parameters and $i \in \{1, 2, 3, 4\}$. For each mini-batch, the four objective gradients are computed separately. PCGrad initializes $g_i^{\text{PC}} = g_i$ and visits the other objectives in a random order. If $(g_i^{\text{PC}})^T g_j < 0$, the conflicting component is removed according to

$$g_i^{\text{PC}} \leftarrow g_i^{\text{PC}} - \frac{(g_i^{\text{PC}})^T g_j}{\|g_j\|_2^2} g_j \quad (14)$$

The projected gradients are finally summed and supplied to the Adam optimizer. Thus, projection is applied to every conflicting pair among MSE, the d-axis partial differential equation (PDE) residual, the q-axis PDE residual, and reciprocity; no objective is assigned a privileged direction in the implemented pairwise mode. The complete pairwise projection and gradient-aggregation procedure is illustrated in Fig. 2.

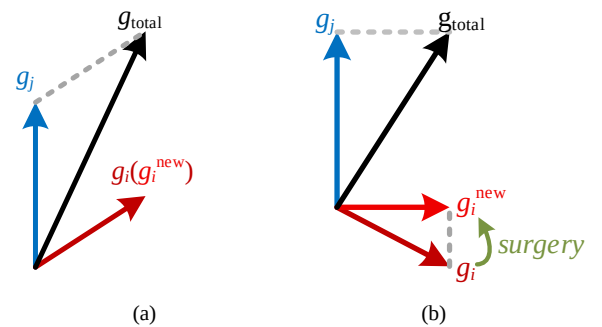


Fig. 2. Schematic diagram of the PCGrad gradient projection reconciliation mechanism. (a) Synergy (angle $\leq 90^\circ$). (b) Conflict & surgery (angle $> 90^\circ$).

V. TRAINING DATA ACQUISITION AND NEURAL NETWORK TRAINING

Having established the theoretical architecture and loss functions of the PINN, acquiring high-quality training samples and formulating a rigorous training procedure are paramount for achieving high-precision identification. Therefore, this section introduces an efficient standstill data acquisition method and investigates the constraining impact of experimental errors on model training.

A. Training Set Acquisition Based on Hysteresis Voltage Injection

This paper employs the hysteresis voltage injection method to acquire standstill training data. This method requires no cumbersome mechanical rotor locking devices or high-precision position sensors, offering high operational convenience. Its fundamental principle involves injecting short-duration voltages into the stator of the motor. When the motor current reaches a predefined limit, the direction of the injected voltage is reversed. The basic voltage injection equation is expressed as:

$$u_{\text{ref}}(k) = \begin{cases} u_{\text{inj}}, & \text{if } i(k) < -I_{\text{max}} \\ -u_{\text{inj}}, & \text{if } i(k) > I_{\text{max}} \\ u_{\text{ref}}(k-1), & \text{otherwise} \end{cases} \quad (15)$$

where u_{inj} is the amplitude of the injected voltage, and I_{max} is the preset current limit. Higher-current standstill injection may increase torque ripple and rotor-displacement risk and must satisfy the electrical and thermal limits of the drive.

By recording the voltage and current data, the flux linkages are subsequently calculated using the discrete integration formulas

$$\begin{cases} \psi_d(k+1) = \psi_d(k) + T_s [u_d(k) - \hat{R}_s i_d(k)] \\ \psi_q(k+1) = \psi_q(k) + T_s [u_q(k) - \hat{R}_s i_q(k)] \end{cases} \quad (16)$$

where T_s is the sampling period, and $\hat{R}_s$ is the estimated stator resistance. To acquire a comprehensive training dataset, two distinct experimental procedures are conducted:

1) *Self-saturation experiment*: Voltage is injected into either the d- or q-axis while the orthogonal-axis current is regulated to zero. The ideal reluctance-torque term is therefore zero. In practice, small residual motion can still arise from initial-alignment and current-regulation errors, so zero rotor displacement is not assumed.

2) *Cross-saturation experiment*: The d- and q-axis voltages are injected simultaneously. The resulting torque ripple can produce oscillatory rotor motion and a small endpoint displacement, although sustained unidirectional rotation is not observed within the retained injection window. The voltage vector must satisfy the dc-link and device limits:

$$u_{\text{dij}}^2 + u_{\text{qij}}^2 \leq \frac{V_{\text{dc}}^2}{3} \quad (17)$$

where V_{dc} is the DC-link voltage; u_{dij} and u_{qij} denote the injected voltages for the d- and q-axes, respectively.

The distribution of the measured trajectory over the target i_d - i_q plane directly determines the magnetic region represented in network training. For an experimental illustration,

Fig. 3 compares two representative cross-saturation current trajectories acquired from the test motor and experimental platform described in Section VI-A. Here, U_d and U_q denote the magnitudes of the d- and q-axis injection-voltage references, respectively. The deliberately extreme equal-amplitude case $U_d = U_q = 130$ V is used to illustrate the consequence of an unfavorable voltage relation. In this acquisition, the d-axis hysteresis period is approximately three times the q-axis period. This near-integer period relation

causes successive trajectory segments to overlap and leaves large regions sparsely traversed. Equal voltage amplitudes are not inherently unsuitable; the selected voltage pair should instead avoid a short repeating period relation in the measured trajectory and provide sufficient traversal of the target current plane.

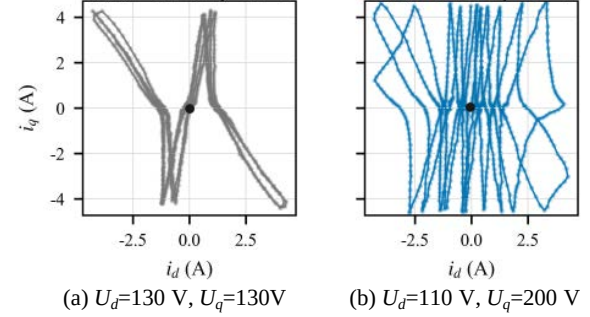


Fig. 3. Experimental cross-saturation current trajectories. (a) Equal d- and q-axis injection amplitudes of 130 V. (b) Selected slow-d/fast-q injection voltages of 110 and 200 V, respectively.

The voltage ratio is estimated from the self-saturation secant inductances. Let $L_{d,\text{sec}}$ and $L_{q,\text{sec}}$ denote representative secant inductances over the target current range, and let ΔI_d and ΔI_q denote the peak-to-peak d- and q-axis current excursions between the corresponding hysteresis limits, respectively. With the average resistive-drop estimates $A_d = R_s I_{d,\text{max}}/2$ and $A_q = R_s I_{q,\text{max}}/2$, the mean current scan rates are approximated as

$$\bar{S}_d \approx \frac{U_d - A_d}{L_{d,\text{sec}}}, \quad \bar{S}_q \approx \frac{U_q - A_q}{L_{q,\text{sec}}} \quad (18)$$

$m = T_d / T_q$ is defined as the target relative number of q-axis traversals during one d-axis traversal, where $T_d \approx \Delta I_d / \bar{S}_d$ and $T_q \approx \Delta I_q / \bar{S}_q$. The voltage relation is then

$$U_q \approx A_q + \kappa (U_d - A_d) \quad (19)$$

where $\kappa = m(\Delta I_q / \Delta I_d)(L_{q,\text{sec}} / L_{d,\text{sec}})$.

Thus, one voltage component can be selected from the available hardware range and the other calculated from the target traversal relation. The resulting pair must satisfy (17) together with the current and device limits, and the measured trajectory should be checked for adequate plane traversal. For the tested motor, an initial engineering range of $m \in [4, 8]$ was considered; $m = 6$ with $U_d = 110$ V gave $U_q = 200$ V and produced the trajectory shown in Fig. 3(b).

While the hysteresis voltage injection method is highly convenient and universally applicable, it inherently possesses certain drawbacks. The voltage integration process is highly susceptible to drift errors, and the high-frequency voltage pulses induce micro-vibrations and slight displacements in the rotor. These factors inevitably introduce errors into the acquired data.

Before each acquisition, a short current-injection prealignment is applied to establish the initial rotor-oriented reference. The hysteresis test then starts from the aligned state without continuous position feedback. The rotor is not mechanically locked during the injection interval; possible displacement is examined in Section V-B.

The usable data window should be selected in complete hysteresis cycles. A single cycle contains both voltage polarities but provides fewer samples and less complete traversal of the current plane, whereas retaining many cycles increases the accumulated flux-integration error. Two complete cycles are therefore recommended as a practical compromise between data coverage and integration drift. Because the experimental comparison is limited to one motor platform, two cycles are presented as a recommended setting rather than a universal optimum. The corresponding physical duration depends on the motor and injection settings.

B. Controlled Simulation, Error Propagation, and Network-size Selection

The Simulink SynRM uses the nonlinear analytical flux-current relation proposed in [21]:

$$\begin{cases} i_d = \left[a_{d0} + a_{dd} |\psi_d|^S + \frac{a_{dq}}{V+2} |\psi_d|^U |\psi_q|^{V+2} \right] \psi_d \\ i_q = \left[a_{q0} + a_{qq} |\psi_q|^T + \frac{a_{dq}}{U+2} |\psi_d|^{U+2} |\psi_q|^V \right] \psi_q \end{cases} \quad (20)$$

where S and T characterize the d- and q-axis self-saturation orders, respectively, while U and V determine the orders of the cross-saturation terms. The two cross-saturation terms are derived from the same magnetic-energy function and therefore satisfy the reciprocity condition. The electrical and mechanical parameters, which are identical to those of the experimental motor, are listed in Table I, while the coefficients, exponents, and numerical settings of the analytical relation are provided in Table II(a). The analytical current-flux grid generated from this relation is used as the simulation ground truth.

TABLE I
ELECTRICAL, MECHANICAL, AND CONTROL PARAMETERS OF
THE SYNRM TEST SYSTEM

Parameter	Value
Rated power/kW	0.55
Pole pair	2
Rated speed/(r/min)	1500
Rated current/A	2.1
Inertia/(kg · m ²)	0.0027
Measured stator resistance/Ω	10.28
Sampling frequency/kHz	10
Switching frequency/kHz	10

Unless otherwise stated, all neural-network training in the simulation and experimental studies uses the same normalization, with $I_{\text{base}} = 4.2144$ A and $\Psi_{\text{base}} = 1.5$ Wb. These values are selected to cover the target current and flux ranges of the identified motor. The common training configuration uses a batch size of 128, a maximum of 1000 epochs, and terminates when the learning rate falls below 10^{-5} . The default model adopts the [6, 4] architecture and PCGrad. Based on the sensitivity analysis in Section VI-B, $\lambda_R = 0.5$ and $\lambda_{\text{PDE}} = 0.1$ are used as the recommended initial values and as the common operating point in the error-propagation and dataset-comparison studies, thereby avoiding

TABLE II
ANALYTICAL MAGNETIC-MODEL SETTINGS, COMBINED-ERROR DEFINITIONS,
AND REPRESENTATIVE ERROR PROPAGATION

TABLE II(A) ANALYTICAL MAGNETIC-MODEL SETTINGS						
Role	a_{d0}	a_{dd}	a_{q0}	a_{qq}	a_{dq}	S, T, U, V
Simulation source model	0.992486	0.266009	3.000650	8.624817	2.862809	5,1,1,0
Analytical baseline (primary dataset)	1.0383	0.2494	0.1942	14.6278	2.7301	5,1,1,0

TABLE II(B) DEFINITIONS OF THE REPRESENTATIVE COMBINED-ERROR CONDITIONS				
Condition	Voltage bias	Current-noise RMS	R_s mismatch	Sampling delay
Mild	+0.5%	0.5% I_{max}	+5%	0.5 T_s
Medium	+1%	1% I_{max}	+5%	0.5 T_s
Severe	+2%	2% I_{max}	+10%	1 T_s

TABLE II(C) REPRESENTATIVE SIMULATION ERROR PROPAGATION		
Case	Label Avg MAE/Wb	Model Avg MAE/Wb
Clean	0.01430	0.01351
+2% voltage bias	0.08384	0.02677
−2% voltage bias	0.08946	0.02016
2% I_{max} current-noise RMS	0.01427±0.00015	0.01824±0.00019
+10% R_s mismatch	0.01844	0.01323
−10% R_s mismatch	0.01723	0.01395
1 T_s delay	0.01385	0.01270
Combined mild	0.02135	0.01540
Combined medium	0.04008	0.02152
Combined severe	0.07730	0.04178

Note: in Table II(a), the simulation source model is used to generate the Simulink data and simulation ground truth. The analytical baseline is fitted only to the primary D130-Q100-D110Q200-2C standstill dataset and is evaluated through numerical inversion in the model comparison of Section VI-B; it is not used to generate the simulation data. In Table II(c), the current-noise entry reports the mean and sample standard deviation over three independently generated noise realizations; the other rows are the listed representative conditions.

condition-specific retuning. The network-size study varies only the hidden-layer dimensions.

For axis $j \in \{d, q\}$, the mean absolute error (MAE_j) and root-mean-square error (RMSE_j) are calculated over the external benchmark points. To avoid singular percentage errors near zero flux, the mean absolute percentage error (MAPE_j) is evaluated only where $|\psi_j| > 5 \times 10^{-3}$ Wb. The average benchmark MAE is defined as $(\text{MAE}_d + \text{MAE}_q)/2$. None of these benchmark points is included in the standstill training set or gradient updates.

The error study comprises one-at-a-time perturbations and three representative combined conditions defined in Table II(b). The individual cases vary voltage-reconstruction bias, current-measurement noise, stator-resistance mismatch, and sampling delay. The combined cases represent mild, medium, and severe error levels. This design identifies the dominant mechanisms and verifies their coupled influence without a full-factorial expansion.

Fig. 4 compares the simulated hysteresis-injection waveforms and flux-integration results under clean and representative nonideal conditions. The enlarged segment in

Fig. 4(c) shows that the integrated q-axis flux progressively departs from the simulation ground truth, particularly under the representative medium combined-error condition. This separation demonstrates the cumulative nature of integration error. An excessively short window limits the number of samples and the current-plane trajectory coverage, whereas extending the window increases the error in the integrated flux labels. Balancing data volume, trajectory coverage, and flux-linkage error, two complete hysteresis cycles are recommended.

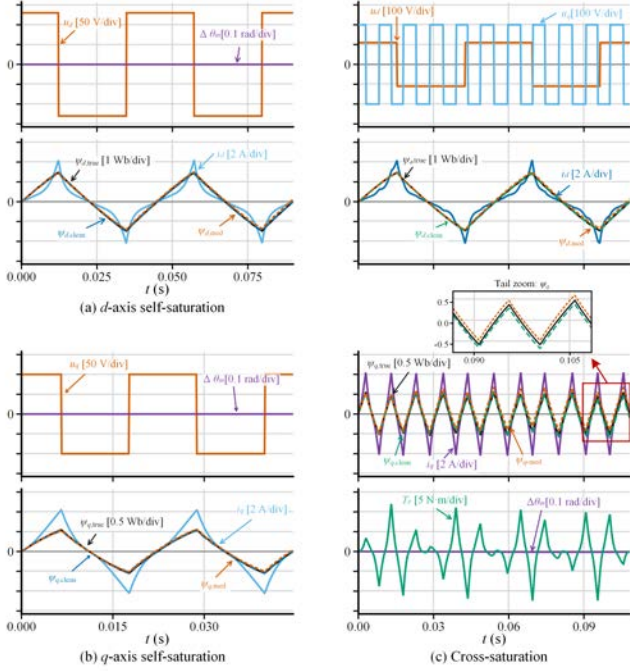


Fig. 4 Simulated hysteresis-injection waveforms. The true flux is plotted as a solid line, and the clean and representative medium combined-error integrals are plotted as dashed lines; the latter is denoted by $\psi_{j,med}$, $j \in \{d, q\}$.

Table II(c) shows that voltage-reconstruction bias is the dominant individual perturbation, the corresponding model average MAEs are 0.02677 and 0.02016 Wb.

At a current-noise root-mean-square (RMS) level of 2% I_{max} , the three-realization label MAE remains close to the clean value, whereas the model MAE increases to 0.01824 ± 0.00019 Wb. A voltage offset contributes a persistent term to the flux integral, while current noise perturbs both the network input coordinates and the resistive-

drop correction. By comparison, the error caused by R_s mismatch reverses with the bidirectional current and can partially cancel over a complete cycle. Sampling misalignment can shift the measured-current and integrated-flux zero crossings relative to one another; however, the $1T_s$ delay case gives a model MAE of 0.01270 Wb, slightly below the clean value, so no accuracy degradation is observed under the present two-cycle condition. This small reduction is attributed to finite-window error cancellation and single-run optimization variability rather than a beneficial effect of delay. The implicit output form enforces $\hat{\psi}_j = 0$ when $i_j = 0$, preventing a learned zero-point offset, but it cannot correct delayed labels. In most listed conditions, the model MAE is lower than the corresponding label MAE, indicating that the physical constraints and global function fitting exhibit some error-smoothing effect. The severe combined condition nevertheless gives the largest model error and approximately triples the clean-model MAE. These observations describe the relative sensitivity in the present short alternating window and do not imply general insensitivity to resistance or delay under other excitation and sampling conditions.

The free-motion simulation retains electromagnetic torque, inertia, friction, and zero external load. The bottom trace of Fig. 4(c) shows the resulting relative mechanical rotor angle and electromagnetic torque using independent oscilloscope-style scales. Over the retained D110Q200 interval, the peak-to-peak mechanical-angle excursion is 3.90×10^{-3} rad, indicating oscillatory motion rather than sustained rotation.

Five candidate architectures are examined using five paired random seeds under the representative medium combined-error condition and are cross-checked on the experimental benchmark. Table III reports the joint comparison. Its simulation column uses the medium combined-error condition defined in Table II(b), whereas its experimental column uses the standstill training data and external benchmark described subsequently in Sections VI-A and V-C, respectively. The [4, 4] network exhibits both a higher mean error and markedly larger seed-to-seed variation. Architectures of [8, 6] and above provide only limited additional accuracy on the experimental benchmark while increasing parameter count and online arithmetic. Considering accuracy, initialization stability, and inference complexity together, [6, 4] is selected as the final network size.

TABLE III
PAIRED MULTI-SEED NETWORK-SIZE COMPARISON AND
FORWARD-PASS COMPLEXITY

Hidden neurons	Medium combined-error simulation MAE /Wb	Experimental benchmark MAE /Wb	Parameters	FP32 weight storage/B	MACs	ELUs
[4,4]	0.02615 $\pm$ 0.00797	0.01707 $\pm$ 0.00696	42	168	32	8
[6,4]	0.02183 $\pm$ 0.00177	0.01286 $\pm$ 0.00195	56	224	44	10
[8,6]	0.02094 $\pm$ 0.00121	0.01283 $\pm$ 0.00074	92	368	76	14
[16,8]	0.02104 $\pm$ 0.00066	0.01278 $\pm$ 0.00069	202	808	176	24
[32,16]	0.01969 $\pm$ 0.00046	0.01264 $\pm$ 0.00024	658	2632	608	48

Note: Values are the mean and sample standard deviation over five paired random seeds. The simulation column uses the medium combined-error condition defined in Table II(b); the experimental column uses the standstill training data and external benchmark described in Sections VI-A and V-C, respectively. FP32 weight storage is calculated using four bytes per parameter. One MAC denotes one multiplication followed by one accumulation; ELU is counted separately.

Training, automatic differentiation, and PCGrad are performed offline, whereas only forward inference is

deployed online. The actual execution time depends on the ELU implementation, compiler, memory placement, and

integration with the control program and is therefore not estimated without device-level measurement. For edge deployment, offline-trained weights, biases, and normalization constants can be exported as static FP32 C arrays. The controller then executes only normalization, the fixed-size dense/ELU forward pass, flux reconstruction, and output scaling once per current-control period. Before closed-loop use, the embedded outputs should be checked against workstation test vectors, and the complete control interrupt should be profiled on the target device to verify memory use and worst-case execution time. Vendor model-to-C tools may be used for MCU families that provide such support.

C. Acquisition of High-precision Test Set Based on the Constant-speed Dynamometer Method

Due to the unavailability of the experimental motor's design blueprints, FEA could not be conducted. To evaluate the training efficacy of the PINN using real-world data, the constant-speed dynamometer method is selected to acquire a high-precision benchmark dataset.

The constant-speed dataset is acquired independently of the standstill injection tests and is excluded from gradient training. It provides an external flux reference for the controlled weight-sensitivity study and for comparing the candidate models under a common criterion; it is not required by the standstill self-commissioning procedure.

In this testbench setup, a load motor drives the SynRM to maintain a constant electrical angular velocity ω_e . Neglecting the current derivative terms, the steady-state flux linkage can be algebraically derived from the stator voltage equations (1):

$$\begin{cases} \psi_d = \frac{u_q - R_s i_q}{\omega_e} \\ \psi_q = -\frac{u_d - R_s i_d}{\omega_e} \end{cases} \quad (21)$$

However, directly applying these equations is highly susceptible to interference from sampling noise, inverter nonlinearities, and thermal variations of stator resistance. Therefore, referencing [16], a multiple compensation strategy is adopted to extract the flux linkage, executed in the following three steps:

1) Mechanical period averaging: Data spanning one complete mechanical period under steady-state conditions is extracted and averaged. This effectively eliminates high-frequency periodic fluctuations induced by current sampling noise and spatial harmonics.

2) Conjugate current compensation: To overcome the impact of thermal variations in stator resistance R_s , conjugate q-axis currents (i.e., i_q and $-i_q$) are applied at the same evaluation point. By averaging the corresponding steady-state voltages, the R_s voltage drop term is mathematically canceled out:

$$\begin{cases} \psi_d = \frac{u_{q,1} - u_{q,2}}{2\omega_e} \\ \psi_q = -\frac{u_{d,1} - u_{d,2}}{2\omega_e} \end{cases} \quad (22)$$

where the subscripts 1 and 2 represent the voltages under positive and negative i_q injections, respectively.

3) Bidirectional rotation averaging: The motor is further operated under both forward and reverse constant-speed conditions. The flux linkage results from both directions are added and averaged to largely eliminate the residual fundamental-wave effects introduced by the inverter.

Through this procedure, an exceptionally high-precision benchmark dataset is established, providing a reliable basis for the absolute error evaluation of the proposed PINN.

VI. EXPERIMENTAL VERIFICATION

This section validates the proposed PINN framework on a 0.55 kW SynRM testbench. The experimental controller applies a current-dependent feedforward compensation for the equivalent inverter error voltage before space-vector pulse-width modulation (SVPWM). This measure reduces inverter-induced voltage distortion, while residual reconstruction errors are represented in the simulation uncertainty study.

A. Experimental Data Acquisition

Fig. 5 illustrates the SynRM experimental testbench. The motor under test is a Siemens 1RV4082B, with its fundamental parameters detailed in Table I. The hardware setup employs a TMS320F28335 digital signal processor (DSP) controller and a commercial inverter. An additional signal conditioning board is integrated for the real-time sampling of the DC-link voltage and phase currents, while a host computer acquires the DSP data via serial communication.

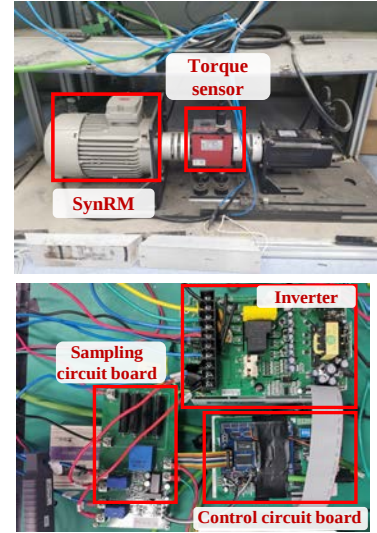


Fig. 5. SynRM experimental test bench.

The primary training dataset, denoted D130-Q100-D110Q200-2C, contains one d-axis self-saturation acquisition at 130 V, one q-axis self-saturation acquisition at 100 V, and one cross-saturation acquisition at $(U_d, U_q) = (110, 200)$ V. Two complete hysteresis cycles are retained from each acquisition, and representative voltage, current, and integrated-flux waveforms under these three injection conditions are shown in Fig. 6. Their overall temporal patterns

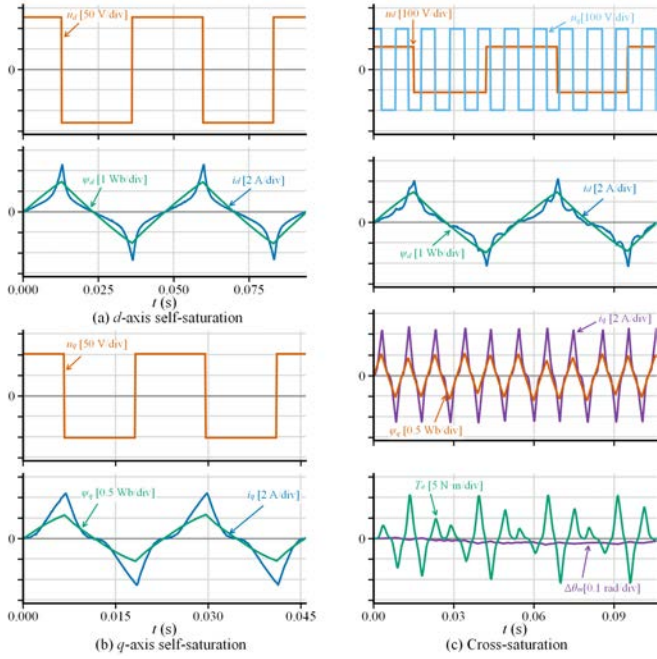


Fig. 6. Experimental hysteresis-injection waveforms under the primary training conditions.

TABLE IV

COMPOSITION AND PURPOSE OF THE REPRESENTATIVE STANDSTILL TRAINING DATASETS

Dataset	d-axis acquisition	q-axis acquisition	Cross acquisition	Pairs	Purpose
D130-Q100-D110Q200-1C	D130, 1C	Q100, 1C	D110Q200, 1C	1227	Cycle comparison
D130-Q100-D110Q200-2C	D130, 2C	Q100, 2C	D110Q200, 2C	2454	Primary dataset
D100-Q100-D110Q200-2C	D100, 2C	Q100, 2C	D110Q200, 2C	2744	d-axis replacement
D160-Q100-D110Q200-2C	D160, 2C	Q100, 2C	D110Q200, 2C	2268	d-axis replacement
D130-Q80-D110Q200-2C	D130, 2C	Q80, 2C	D110Q200, 2C	2590	q-axis replacement
D130-Q90-D110Q200-2C	D130, 2C	Q90, 2C	D110Q200, 2C	2522	q-axis replacement
D130-Q110-D110Q200-2C	D130, 2C	Q110, 2C	D110Q200, 2C	2410	q-axis replacement
D130-Q100-D90Q210-2C	D130, 2C	Q100, 2C	D90Q210, 2C	2710	Cross-voltage replacement
D130-Q100-D140Q180-2C	D130, 2C	Q100, 2C	D140Q180, 2C	2230	Cross-voltage replacement
Dall-Qall-D110Q200-2C	All tested d-axis, 2C	All tested q-axis, 2C	D110Q200, 2C	5968	Merged self-saturation
D130-Q100-CrossAll-1C	D130, 1C	Q100, 1C	All tested cross cases, 1C	2289	Merged one-cycle cross-saturation
D130-Q100-CrossAll-2C	D130, 2C	Q100, 2C	All tested cross cases, 2C	4602	Merged cross-saturation
Dall-Qall-CrossAll-1C	All tested d-axis, 1C	All tested q-axis, 1C	All tested cross cases, 1C	4025	All one-cycle short windows
Dall-Qall-CrossAll-2C	All tested d-axis, 2C	All tested q-axis, 2C	All tested cross cases, 2C	8116	All two-cycle short windows

Note: "Dxxx", "Qxxx", and "DxxxQxxx" denote the d-axis, q-axis, and cross-saturation voltage-injection conditions, respectively; "1C" and "2C" indicate that one and two complete hysteresis cycles are retained, respectively.

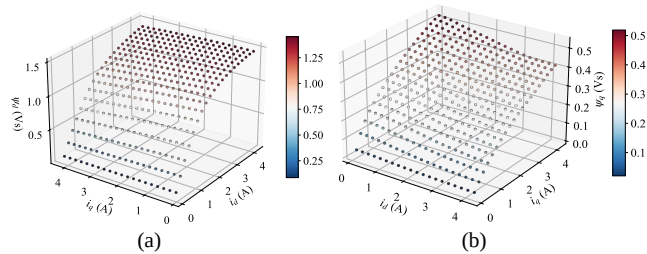


Fig. 7. The d- and q-axis flux linkage data obtained using the constant-speed dynamometer method. (a) d-axis flux linkage data. (b) q-axis flux linkage data.

subsequent fitting evaluations. The benchmark in Fig. 7 is not included in any standstill training dataset or gradient update and provides the common external reference for the evaluations below.

B. Benchmark Comparison, Ablation, and Architecture Selection

The conventional ANN, the structure-constrained network

are consistent with the corresponding simulated waveforms in Fig. 4. The additional datasets in Table IV replace one injection voltage at a time or merge several short acquisitions. This construction separates the effects of cycle number, self-saturation voltage, cross-saturation voltage, and multi-window aggregation.

As shown in the bottom trace of Fig. 6(c), the calibrated relative mechanical rotor angle remains oscillatory during the retained two-cycle D110Q200 interval. Its peak-to-peak excursion is 0.0340 rad, and its endpoint displacement is -0.0113 rad, rather than indicating sustained unidirectional rotation. Within this observed displacement range, the complete-cycle data still provide convergent three-seed training and finite external-benchmark errors in Section VI-B, indicating that the bounded motion does not prevent stable acquisition and training under the tested conditions.

Furthermore, the constant-speed dynamometer method is utilized to obtain a high-precision test dataset. A point-by-point sweep is conducted within the current range of $[0, 2$ per unit (p.u.)] with a step size of 0.1 p.u. Ultimately, an exceptionally comprehensive external high-precision flux matrix is constructed (Fig. 7), serving as the benchmark for

(SCN), the proposed PINN, the analytical flux-current model, and a direct third-order polynomial model are evaluated on the same constant-speed benchmark. All neural models follow the normalization, batch-size, and stopping settings specified in Section V-B. The sensitivity and ablation experiments intentionally vary the indicated physics weights or gradient-handling method; otherwise, the common training configuration is retained. Unless otherwise stated, repeated neural experiments report the mean and sample standard deviation over seeds 21, 42, and 84. For spatial comparison, the representative neural-network error maps consistently use seed 21 so that each panel corresponds to one actual model rather than an average assembled from different predictions.

For the main model comparison, the nonlinear analytical relation introduced in Section V-B is fitted to the primary D130-Q100-D110Q200-2C standstill dataset and retained as an analytical reference model. Its dataset-specific coefficients are listed in Table II(a). Because the model maps flux linkage

to current, evaluating flux linkage from a current command requires numerical nonlinear inversion. A separate traditional polynomial baseline maps the normalized dq currents directly to each flux component using all monomials up to the total degree three.

Table V(a), Figs. 8 and 9 compare the five methods on the common external benchmark. Fig. 8 shows the fitted d- and q-axis flux-linkage surfaces, whereas Fig. 9 shows their absolute-error distributions. The proposed PINN gives the lowest average error among the compared methods: Its three-seed average MAE is 0.01248 ± 0.00254 Wb, which is 27.5% lower than that of the analytical flux-current model (0.01721 Wb) and 38.5% lower than that of the SCN (0.02028 ± 0.00168 Wb). The analytical model nevertheless remains a strong reference and outperforms the SCN and direct polynomial baselines. From an engineering perspective, the analytical model has a compact parameterization, but current-to-flux evaluation requires iterative nonlinear inversion. The proposed network directly outputs flux linkage without inversion. The spatial plots show where each representation departs from the benchmark instead of treating any reference as uniformly inferior.

TABLE V
EXTERNAL-BENCHMARK COMPARISON

TABLE V(A) CANDIDATE-MODEL COMPARISON

Method	MAE _d /Wb	MAE _q /Wb	Average benchmark MAE/Wb
ANN	0.04074	0.04430	0.04252±0.00265
SCN	0.01861	0.02196	0.02028±0.00168
Analytical flux-current model	0.01752	0.01690	0.01721
Traditional polynomial	0.10342	0.04346	0.07344
Proposed PINN	0.01192	0.01303	0.01248±0.00254

TABLE V(B) PHYSICS-OBJECTIVE AND PCGRAD ABLATIONS

Active objectives	λ_R	λ_{PDE}	Gradient handling	Average benchmark MAE/Wb
MSE	0	0	Single objective	0.02028±0.00168
MSE+PDE	0	0.1	Pairwise PCGrad	0.01836±0.00025
MSE+reciprocity	0.5	0	Pairwise PCGrad	0.01362±0.00203
MSE+reciprocity+PDE	0.5	0.1	Direct sum	0.01510±0.00224
MSE+reciprocity+PDE	0.5	0.1	Pairwise PCGrad	0.01248±0.00254

With the MSE weight fixed at unity, the two physics weights are first examined independently by disabling the other physics term, and every setting is repeated using the paired seeds 21, 42, and 84. Fig. 10(a) shows that reciprocity-only training improves on MSE-only training throughout the tested range; its mean MAE decreases from 0.01455 Wb at $\lambda_R = 0.05$ to 0.01362 Wb at 0.5 , and then increases to 0.01401 Wb at 1.0 . For the PDE-only models, the mean error decreases through $\lambda_{PDE} = 0.05$, increases slightly at 0.1 , and decreases again to 0.01780 Wb at 0.2 . This non-monotonic response shows that an independently favorable PDE weight does not determine its value in the combined objective. The two objectives are then combined in the 3×3 local interaction study in Fig. 10(b), again using the same three seeds. Compared with $\lambda_{PDE} = 0.1$, setting $\lambda_{PDE} = 0.2$ increases the mean error at all three tested reciprocity weights and produces marked seed-to-seed variation when reciprocity is weak. Increasing λ_R within the tested local range has a smaller effect and partly mitigates this degradation. The differences between $(\lambda_R, \lambda_{PDE}) = (0.5, 0.1)$ and the neighboring low-error settings are small relative to the seed-to-seed variation. Taken together, the independent and interaction results provide the principal basis for selecting $(0.5, 0.1)$ as the common operating point throughout the simulation and experimental studies, except when the corresponding factor is intentionally varied in sensitivity or ablation tests.

The pre-projection diagnostics in Fig. 10(c) show frequent conflicts among all three objective groups. PDE-reciprocity has the highest aggregated conflict rate, while MSE-reciprocity conflicts are similarly frequent. These observations are consistent with the non-monotonic weight interaction and motivate using PCGrad. These diagnostics characterize the degree of optimization interference, while predictive accuracy is assessed independently on the benchmark. The weights determine the gradient magnitudes entering the multi-objective update, whereas PCGrad removes only the conflicting components; reasonable loss scaling therefore remains necessary.

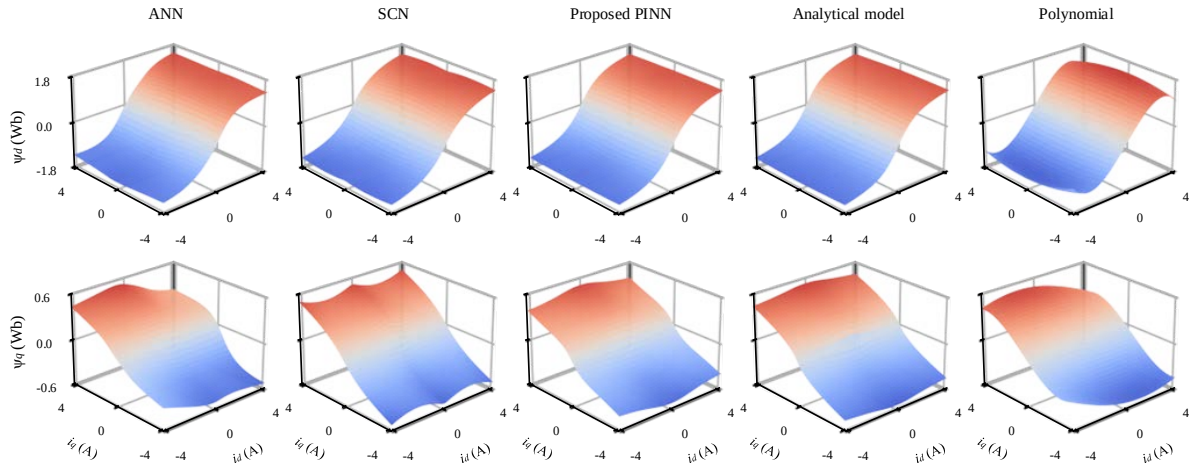


Fig. 8. Fitted d- and q-axis flux-linkage surfaces of the ANN, SCN, proposed PINN, analytical flux-current model, and direct third-order polynomial model. All models use the same standstill training dataset, and the neural-network panels correspond to the common seed-21 setting.

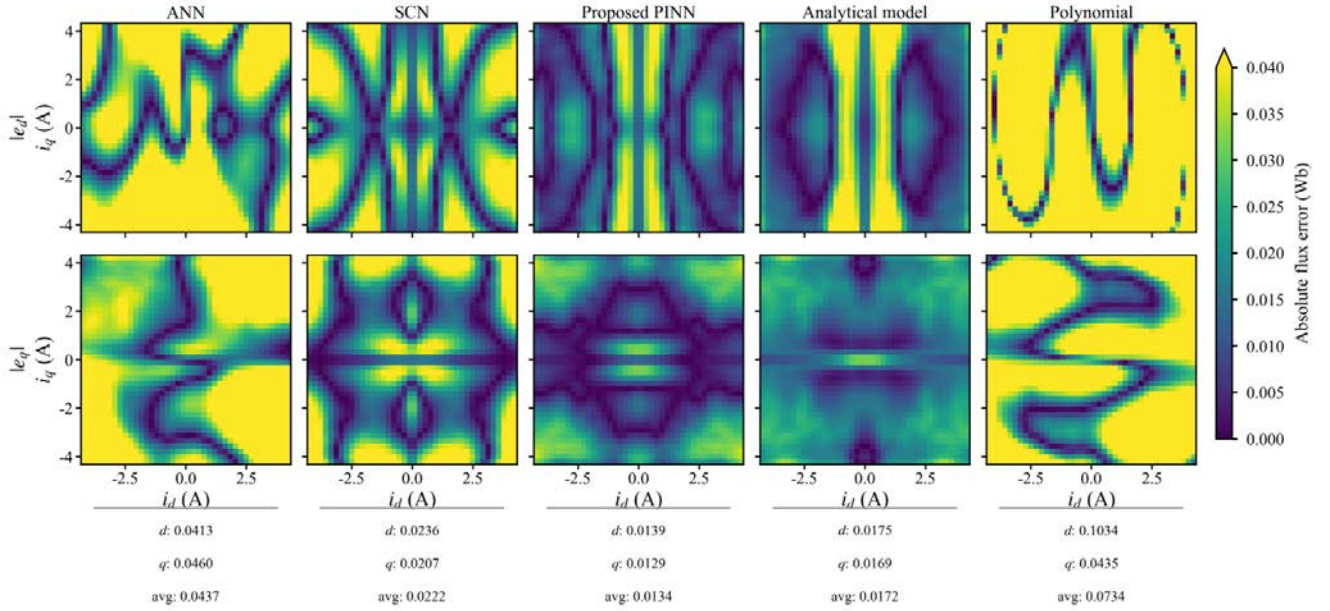


Fig. 9. Flux-linkage absolute-error maps of the five models evaluated on the external constant-speed benchmark for the common seed-21 setting. The values below each column give the d-axis, q-axis, and average MAEs in Wb. Three-seed statistics are reported in Table V(a).

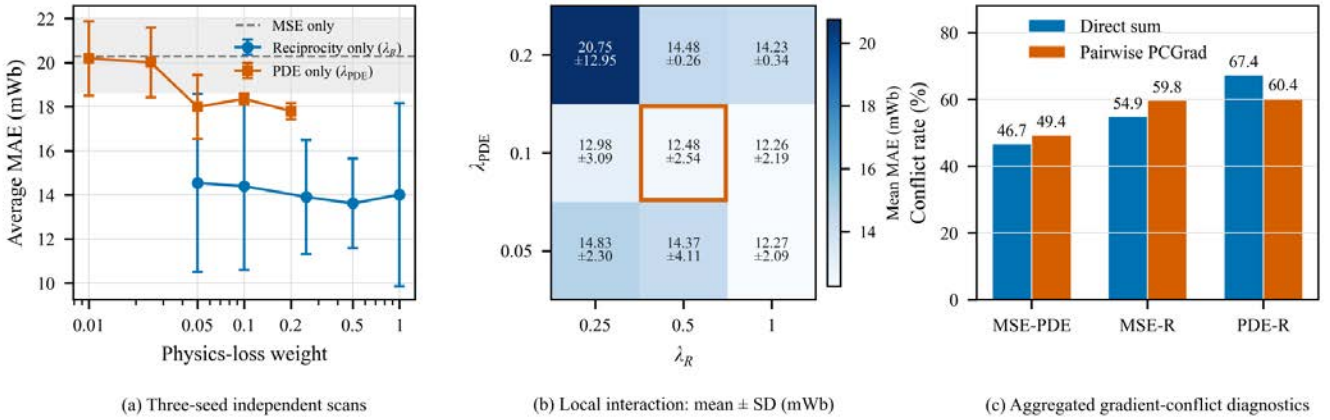


Fig. 10. Weight sensitivity and optimization diagnostics. (a) Three-seed mean and sample standard deviation for the independent reciprocity- and voltage-equation-weight scans with the other physics term disabled. (b) Three-seed mean and sample standard deviation for their local interaction. (c) Pre-projection gradient conflicts grouped into MSE-PDE, MSE-reciprocity, and PDE-reciprocity categories.

The ablation results are summarized in Table V(b), where the active physics weights and gradient-handling method are stated explicitly. Relative to MSE-only training 0.02028 ± 0.00168 Wb, reciprocity reduces the mean MAE by 32.8%; adding the voltage-equation residual reduces it by a further 8.4% to 0.01248 ± 0.00254 Wb. With the complete objective set, PCGrad reduces the mean MAE by 17.4% relative to direct gradient summation (0.01510 ± 0.00224 Wb). PCGrad operates pairwise on the four nonzero weighted objectives, namely MSE, the two axis-wise voltage-equation residuals, and reciprocity.

As reported earlier in Table III, the [4, 4] network has both higher mean error and larger seed-to-seed variation. On the experimental benchmark, increasing the architecture from [6, 4] to [8, 6] changes the mean MAE only from 0.01286 to 0.01283 Wb, while increasing the MAC count from 44 to 76. Considering benchmark accuracy, seed-to-seed stability, parameter count, and floating-point operation count, the [6, 4]

architecture is retained for the subsequent experiments.

Table VI(a) separates the duration of each integration window from the number of independently acquired windows. For the primary dataset, retaining two complete cycles reduces the three-seed average MAE from 0.02649 to 0.01248 Wb (52.9%), providing experimental support for the preceding two-cycle recommendation. More importantly, merging several independently integrated one-cycle acquisitions substantially improves the result over the single-acquisition one-cycle dataset and approaches the corresponding two-cycle results without extending any individual integration interval, at the cost of additional acquisition runs. For example, the merged one-cycle dataset reaches 0.01148 ± 0.00138 Wb, close to the corresponding two-cycle result of 0.01098 ± 0.00147 Wb. These results also show that multiple independently integrated short acquisitions can increase trajectory diversity without relying on a long continuous integration window.

TABLE VI
EFFECTS OF COMPLETE-CYCLE WINDOWS, INJECTION VOLTAGES, AND
MULTI-WINDOW TRAINING DATA

TABLE VI(A) COMPLETE-CYCLE AND MULTI-WINDOW COMPARISON USING
THREE RANDOM SEEDS

Dataset	Pairs	Average benchmark MAE/Wb
D130-Q100-D110Q200-1C	1227	0.02649±0.01193
D130-Q100-D110Q200-2C	2454	0.01248±0.00254
D130-Q100-CrossAll-1C	2289	0.01262±0.00149
D130-Q100-CrossAll-2C	4602	0.01199±0.00216
Dall-Qall-CrossAll-1C	4025	0.01148±0.00138
Dall-Qall-CrossAll-2C	8116	0.01098±0.00147

TABLE VI(B) INJECTION-VOLTAGE REPLACEMENT USING
THREE RANDOM SEEDS

Dataset	Pairs	Average benchmark MAE/Wb
D130-Q100-D110Q200-2C	2454	0.01248±0.00254
D100-Q100-D110Q200-2C	2744	0.01431±0.00209
D160-Q100-D110Q200-2C	2268	0.01488±0.00261
D130-Q80-D110Q200-2C	2590	0.01367±0.00116
D130-Q90-D110Q200-2C	2522	0.01500±0.00199
D130-Q110-D110Q200-2C	2410	0.01371±0.00177
D130-Q100-D90Q210-2C	2710	0.01300±0.00174
D130-Q100-D140Q180-2C	2230	0.01931±0.00189

TABLE VI(C) MERGED SELF-SATURATION
COMPARISON USING THREE RANDOM SEEDS

Dataset	Pairs	Average benchmark MAE/Wb
D130-Q100-D110Q200-2C	2454	0.01248±0.00254
Dall-Qall-D110Q200-2C	5968	0.01331±0.00421

Note: Neural-network results are reported as the mean and sample standard deviation over seeds 21, 42, and 84.

Under the fixed training configuration, each tested self-saturation voltage produces a usable model, although the replacement datasets span average MAEs of 0.01300–0.01931 Wb, as summarized in Table VI(b). The self-saturation tests establish the axis boundaries and secant-inductance trends; reaching the target current range is necessary, while the resulting trajectory distribution and integrated-label quality still affect the final accuracy. Among the three tested single cross-saturation acquisitions, D110Q200 gives the lowest three-seed mean error under the common weights. Table VI(c) further shows that merging all self-saturation acquisitions changes the result from 0.01248±0.00254 to 0.01331±0.00421 Wb when the cross-saturation acquisition is fixed. In contrast, merging all independently integrated short acquisitions gives the lowest mean error in Table VI(a). Thus, additional short windows are useful when they add complementary trajectory coverage, but benchmark error is not determined by sample count alone. Injection design should consider both the traversed operating region and the quality of the integrated labels.

These results demonstrate robustness to several training-data compositions for the tested motor under a common normalization and loss setting. Because all weight studies use the scaling bases specified in Section V-B, the reported physics-loss coefficients should be interpreted as recommended initial values under this normalization rather

than as motor-independent optima. For a new motor, the scaling bases should first be adapted to its operating envelope, after which the reported coefficients can be used as initial values. Held-out injection-label error, reciprocity and voltage-equation residuals, convergence behavior, and gradient-conflict statistics can reject unstable or physically inconsistent settings, but independent reference data or application-level tests are still required to verify absolute flux accuracy.

VII. CONCLUSION

This paper presented a standstill flux-linkage identification framework for SynRMs based on an implicit secant-inductance network and multi-physics gradient projection. The architecture embeds the zero-flux and parity constraints, while Maxwell reciprocity and the discrete voltage equation act as complementary soft objectives. Ablation results on a common external benchmark verify the contributions of the two physics objectives and PCGrad. Controlled simulations show the accumulation of voltage-integration error and quantify rotor movement under free mechanical dynamics. The acquisition procedure recommends two complete hysteresis cycles as a compromise between trajectory coverage and integration drift and determines the cross-saturation voltage ratio from self-saturation secant inductances. Comparisons using different injection data show that trajectory range and label quality are more informative than sample count alone. A paired multi-seed study supports the [6, 4] network as a compact accuracy-complexity tradeoff for per-cycle inference. The standstill training-data acquisition requires neither a mechanical rotor-locking device nor a dynamometer; the constant-speed data in this study are excluded from gradient training and used as an external research reference. The reported loss weights and two-cycle window were validated on one motor platform; their transferability to motors with different ratings and saturation characteristics remains to be investigated. Moreover, self-consistency checks without a flux reference cannot certify absolute mapping accuracy. Future work will investigate cross-motor transfer and evaluate the practical effectiveness of the proposed method in sensorless control and other real-time control algorithms.

REFERENCES

- [1] M. Murataliyev, M. Degano, and M. Di Nardo *et al.*, “Synchronous Reluctance Machines: a Comprehensive Review and Technology Comparison,” *Proc. of the IEEE*, vol. 110, no. 3, pp. 382–399, Mar. 2022.
- [2] X. Li, Y. W. Wang, and Y. H. Cheng *et al.*, “An Overview of High-efficiency Synchronous Reluctance Machines,” *CES Trans. on Elect. Mach. and Syst.*, vol. 7, no. 1, pp. 81–91, Mar. 2023.
- [3] C. W. Diao, W. L. Zhao, and Y. Liu *et al.*, “Permanent Magnet Assisted Synchronous Reluctance Motor with Asymmetric Rotor for High Torque Performance,” *CES Trans. on Elect. Mach. and Syst.*, vol. 7, no. 2, pp. 179–186, Jun. 2023.
- [4] N. Bianchi, S. Bolognani, and E. Carraro *et al.*, “Electric Vehicle Traction based on Synchronous Reluctance Motors,” *IEEE Trans. on Ind. Appl.*, vol. 52, no. 6, pp. 4762–4769, Nov.-Dec. 2016.
- [5] T. Suzuki, T. Kojima, and H. Yamasaki *et al.*, “Experimental Verification of Position Sensorless Control of SynRMs based on

- Magnetic Saturation Depending on Current Phase Angles,” in *Proc. of 2019 IEEE Int. Conf. on Ind. Technol. (ICIT)*, Melbourne, VIC, Australia, Feb. 2019, pp. 255–260.
- [6] C. Bao, S. J. Song, and H. D. Chen *et al.*, “High-precision Virtual Signal Injection MTPA Control of Synchronous Reluctance Motor based on Parameter Identification Considering Electromagnetic Torque Partial Derivation Term,” *IEEE Trans. on Transp. Electrification*, vol. 11, no. 4, pp. 10453–10460, Aug. 2025.
- [7] A. Accetta, M. Cirrincione, and M. C. Di Piazza *et al.*, “Analytical Formulation of a Maximum Torque Per Ampere (MTPA) Technique for SynRMs Considering the Magnetic Saturation,” *IEEE Trans. on Ind. Appl.*, vol. 56, no. 4, pp. 3846–3854, Jul.-Aug. 2020.
- [8] B. J. Li, J. B. Zou, and Y. X. Xu *et al.*, “Sensorless Control of DTP-SynRM with Hybrid Flux Observer and Disturbance Observer Considering Magnetic Saturation and Cross-coupling Effect,” *IEEE Trans. on Power Electron.*, vol. 40, no. 9, pp. 12295–12308, Sept. 2025.
- [9] P. C. Du, B. Wang, and D. G. Xu, “Parameter Robustness Sensorless SynRM Drives via LUT-driven Position Error Compensation for Nonlinear Flux Observer,” *IEEE Trans. on Ind. Electron.*, vol. 73, no. 4, pp. 5078–5090, Apr. 2026.
- [10] L. P. Yang, Y. T. Lu, and G. D. Feng *et al.*, “Multiparameter Estimation of PMSM Combining Mechanical and Electrical Models with Nonlinear Saturated Inductance and Voltage Fitting,” *CES Trans. on Elect. Mach. and Syst.*, vol. 10, no. 1, pp. 109–120, Mar. 2026.
- [11] A. Vagati, M. Pastorelli, and G. Franceschini, “High-performance Control of Synchronous Reluctance Motors,” *IEEE Trans. on Ind. Appl.*, vol. 33, no. 4, pp. 983–991, Jul.-Aug. 1997.
- [12] R. Delpoux, T. Huguet, and F. B. Argomedo *et al.*, “Finite Element Dq-model for MTPA Flux Control of Synchronous Reluctance Motor (SynRM),” in *Proc. of 2023 IEEE 32nd Int. Symp. on Ind. Electron. (ISIE)*, Helsinki, Finland, Jun. 2023, pp. 1–6.
- [13] S. A. Odhano, P. Pescetto, and H. A. A. Awan *et al.*, “Parameter Identification and Self-commissioning in AC Motor Drives: a Technology Status Review,” *IEEE Trans. on Power Electron.*, vol. 34, no. 4, pp. 3603–3614, Apr. 2019.
- [14] K. M. Rahman, and S. Hiti, “Identification of Machine Parameters of a Synchronous Motor,” *IEEE Trans. on Ind. Appl.*, vol. 41, no. 2, pp. 557–565, Mar.-Apr. 2005.
- [15] K. Liu, J. H. Feng, and S. Y. Guo *et al.*, “Identification of Flux Linkage Map of Permanent Magnet Synchronous Machines Under Uncertain Circuit Resistance and Inverter Nonlinearity,” *IEEE Trans. on Ind. Informat.*, vol. 14, no. 2, pp. 556–568, Feb. 2018.
- [16] E. Armando, R. I. Bojoi, and P. Guglielmi *et al.*, “Experimental Identification of the Magnetic Model of Synchronous Machines,” *IEEE Trans. on Ind. Appl.*, vol. 49, no. 5, pp. 2116–2125, Sept.-Oct. 2013.
- [17] G. Pellegrino, B. Boazzo, and T. M. Jahns, “Magnetic Model Self-identification for PM Synchronous Machine Drives,” *IEEE Trans. on Ind. Appl.*, vol. 51, no. 3, pp. 2246–2254, May-Jun. 2015.
- [18] B. Stumberger, G. Stumberger, and D. Dolinar *et al.*, “Evaluation of Saturation and Cross-magnetization Effects in Interior Permanent-magnet Synchronous Motor,” *IEEE Trans. on Ind. Appl.*, vol. 39, no. 5, pp. 1264–1271, Sept.-Oct. 2003.
- [19] S. A. Odhano, P. Giangrande, and R. I. Bojoi *et al.*, “Self-commissioning of Interior Permanent- Magnet Synchronous Motor Drives with High-frequency Current Injection,” *IEEE Trans. on Ind. Appl.*, vol. 50, no. 5, pp. 3295–3303, Sept.-Oct. 2014.
- [20] N. Bedetti, S. Calligaro, and R. Petrella, “Stand-still Self-identification of Flux Characteristics for Synchronous Reluctance Machines Using Novel Saturation Approximating Function and Multiple Linear Regression,” *IEEE Trans. on Ind. Appl.*, vol. 52, pp. 3083–3092, 2016.
- [21] M. Hinkkanen, P. Pescetto, and E. Mölsä *et al.*, “Sensorless Self-commissioning of Synchronous Reluctance Motors at Standstill without Rotor Locking,” *IEEE Trans. on Ind. Appl.*, vol. 53, no. 3, pp. 2120–2129, May-Jun. 2017.
- [22] T. G. Woo, S. W. Park, and S. C. Choi *et al.*, “Flux Saturation Model Including Cross Saturation for Synchronous Reluctance Machines and Its Identification Method at Standstill,” *IEEE Trans. on Ind. Electron.*, vol. 70, no. 3, pp. 2318–2328, Mar. 2023.
- [23] Q. W. Wang, N. N. Zhao, and G. L. Wang *et al.*, “An Offline Parameter Self-learning Method Considering Inverter Nonlinearity with Zero-axis Voltage,” *IEEE Trans. on Power Electron.*, vol. 36, no. 12, pp. 14098–14109, Dec. 2021.
- [24] J. H. Lee, Y. J. Lee, and M. S. Lee *et al.*, “Estimation of Flux Saturation Model for SynRMs Using Artificial Neural Network,” *IEEE Trans. on Ind. Appl.*, vol. 61, no. 2, pp. 3143–3151, Mar.-Apr. 2025.
- [25] M. R. Raia, M. Ruba, and R. O. Nemes *et al.*, “Artificial Neural Network and Data Dimensionality Reduction based on Machine Learning Methods for PMSM Model Order Reduction,” *IEEE Access*, vol. 9, pp. 102345–102354, Jul. 2021.
- [26] C. Ye, Y. K. Jia, and Y. S. Song *et al.*, “Estimation of Flux Saturation Model for SynRMs Using Physics-informed Neural Networks at Standstill,” in *Proc. of 2025 IEEE 3rd Int. Power Electron. and Appl. Symp. (PEAS)*, Shenzhen, China, Nov. 2025, pp. 602–608.
- [27] KrisCCeng. (2026). SynRM-PINN. GitHub. [Online]. Available: <https://github.com/KrisCCeng/SynRM-PINN.git>
- [28] G. E. Karniadakis, I. G. Kevrekidis, and L. Lu *et al.*, “Physics-informed Machine Learning,” *Nat. Rev. Phys.*, vol. 3, pp. 422–440, May 2021.
- [29] M. Raissi, P. Perdikaris, and G. E. Karniadakis, “Physics-informed Neural Networks: a Deep Learning Framework for Solving Forward and Inverse Problems Involving Nonlinear Partial Differential Equations,” *J. of Comput. Phys.*, vol. 378, pp. 686–707, Feb. 2019.
- [30] S. Son, H. Lee, and D. Jeong *et al.*, “A Novel Physics-informed Neural Network for Modeling Electromagnetism of a Permanent Magnet Synchronous Motor,” *Adv. Eng. Inform.*, vol. 57, pp. 102035, Aug. 2023.
- [31] R. Cipolla, Y. Gal, and A. Kendall, “Multi-task Learning Using Uncertainty to Weigh Losses for Scene Geometry and Semantics,” in *Proc. of 2018 IEEE/CVF Conf. on Comput. Vis. and Pattern Recognit.*, Salt Lake City, UT, USA, Jun. 2018, pp. 7482–7491.
- [32] L. D. McClenny, and U. M. Braga-Neto, “Self-adaptive Physics-informed Neural Networks,” *J. of Comput. Phys.*, vol. 474, pp. 111722, Feb. 2023.
- [33] T. H. Yu, S. Kumar, and A. Gupta *et al.*, “Gradient Surgery for Multi-task Learning,” in *Adv. Neural Inf. Process. Syst.*, vol. 33, 2020, pp. 5824–5836.



Cheng Ye (Student Member, IEEE) received the B.S. degree in electrical engineering and automation from the Harbin Institute of Technology, Harbin, China, in 2024. He is currently pursuing the M.S. degree in electrical engineering at Xi'an Jiaotong University, Xi'an, China.

His current research interests include physics-informed neural networks (PINNs), synchronous motor parallel systems, and motor drives.



Yankai Jia received the B.S. degree in electrical engineering and automation from Qingdao University, Qingdao, China, in 2024. He is currently pursuing the M.S. degree in electrical engineering at Xi'an Jiaotong University, Xi'an, China.

His research interests include electromagnetic compatibility, electric motor control, power electronics, and EMI suppression in motor drive systems.



Yusheng Song received the B.S. degree in electrical engineering and automation from Northeastern University, Shenyang, China, in 2023, and received the M.S. degree in electrical engineering from Xi'an Jiaotong University, Xi'an, China, in 2026.

His research interests include dual inverters, motor drive systems, and power conversion technologies.



Changxing Guo received the B.S. degree in electrical engineering and automation from the Harbin Institute of Technology, Harbin, China, in 2023, and received the M.S. degree in electrical engineering from Xi'an Jiaotong University, Xi'an, China, in 2026.

His current research interests include the control and parameter identification of permanent magnet synchronous motors (PMSMs) and the application of physics-informed neural networks (PINNs) in motor modeling.



Chuanwen Shen received the M.S. degree in control theory and the Ph.D. degree in electrical machines from Xi'an Jiaotong University, Xi'an, China, in 1991 and 1997, respectively.

In 1997, he joined the School of Electrical Engineering at Xi'an Jiaotong University as a Lecturer, where he has been an Associate Professor of industrial automation since 1999. In 2002, he was a Visiting Scholar with the University of Toronto, Toronto, ON, Canada, for one year.

His current research interests include electric drives, alternating current (ac) speed regulation, and inverter development.