

Design and Comparative Analysis of Permanent Magnet Vernier Motors for High-torque Performance Robotic Joints

Rui Li, *Graduate Student Member, IEEE*, Daohan Wang, *Senior Member, IEEE*, Yingsa Li, *Graduate Student Member, IEEE*, Dandan Song, Ran Zhang, and Xiuhe Wang, *Member, IEEE*

Abstract—High-torque-density actuators are critical for enhancing the dynamic performance of articulated robotic systems. This paper presents the design and analysis of a high-performance 62-pole 12-slot (62p12s) permanent magnet vernier motor (PMVM) featuring a triple-split-tooth topology, specifically optimized for robotic joint applications. To maximize torque capability while suppressing torque ripple, a multi-objective optimization framework is implemented to determine the optimal split-tooth geometries and pole-slot configurations. The electromagnetic characteristics—including air-gap magnetic flux density, self-inductance, and power factor—are analytically formulated and subsequently validated through finite element analysis (FEA). To verify the proposed design, a prototype was fabricated and benchmarked against a commercial 42-pole 36-slot (42p36s) permanent magnet synchronous motor (PMSM) with the same overall dimensions and electrical loading. Experimental results, encompassing back electromotive force (back-EMF), static/instantaneous torque, and efficiency, demonstrate that the proposed PMVM achieves a 24.2% increase in torque density and a 6.6% reduction in torque ripple compared to the benchmark PMSM. This study underscores the potential of PMVMs as a high-torque-density solution for advanced robotic articulation.

Index Terms—Permanent magnet vernier motor (PMVM), Permanent magnet synchronous motor (PMSM), Robotic joint actuation, Torque performance.

I. INTRODUCTION

IN recent years, robotics has witnessed significant technological evolution across industrial and medical domains. The actuation methodology of robotic systems

critically determines their capacity to execute preprogrammed maneuvers with precision [1]. Electromechanical drive systems have emerged as the dominant solution for modern robotics, offering superior power-to-mass ratios, exceptional dynamic performance, and precise motion-control capabilities when compared with hydraulic and pneumatic actuation systems [2]–[4].

As critical electromechanical components in electric robotic systems, motor topology selection and actuation methodology critically influence system performance, cost-effectiveness, and operational reliability [5]. The dynamic operating conditions of robotic systems—characterized by cyclic start-stop operation and intermittent overload conditions—have positioned torque-density optimization in robotic joint motors as a critical research frontier. Among the motor topologies, permanent magnet synchronous motors (PMSMs) have emerged as the dominant solution for robotic joint systems, owing to their superior torque density, compact form factor, and high operational efficiency [6]–[7]. Conventional PMSMs with integer-slot distributed windings (ISDW) exhibit substantial cogging torque and torque ripple, compromising motion positioning accuracy—particularly during low-speed operation [8]–[9]. Fractional-slot concentrated winding (FSCW) PMSMs mitigate these limitations through reduced end-winding length and compact axial design [10]–[11]. However, elevated air-gap harmonic content excites low-order radial electromagnetic forces, inducing detrimental vibration and acoustic noise. Outer-rotor PMSMs exhibit smoother torque output and improved convective cooling performance compared to their inner-rotor counterparts [12]–[13]. In [14], a specialized forced air-cooling structure was developed for high-power-density external-rotor surface-mount PMSMs, and the cooling system's performance was rigorously verified by an experimental prototype under varied loading conditions. As noted in [15], the adoption of Halbach arrays in PMSMs is particularly effective for robotic actuation; by improving the structure of the motor, the output torque has been increased and torque ripple reduced.

Permanent magnet vernier motors (PMVMs) have garnered significant research interest owing to their inherent high-torque-density characteristics and structural topological simplicity [16]–[18]. A dual-side PMVM proposed in [19]

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Rui Li, Daohan Wang, Yingsa Li, Dandan Song, and Xiuhe Wang are with the School of Electrical Engineering, Shandong University, Jinan 250100, China (e-mail: 202434682@mail.sdu.edu.cn; dhwang@sdu.edu.cn; 202420767@mail.sdu.edu.cn; songdandan@mail.sdu.edu.cn; wangxhpmms@163.com).

Ran Zhang is with School of Automation and Electrical Engineering, Shandong University of Aeronautics, Binzhou 256600, China (e-mail: zhangran@mail.bzu.edu.cn).

(Corresponding author: Daohan Wang)

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enhances torque characteristics through harmonic suppression of torque-ripple-inducing components and selective amplification of torque-producing harmonics. A design is presented for a high-speed, high-power motor for a four-legged robot actuator that is optimized using the weighted sum method based on the Taguchi method and the response surface method in [20]. In [21], a design method for improving the power factor of PMVMs is proposed to address the issue of their low power factor. However, current literature exhibits a notable absence of systematic comparative analyses between PMSMs and PMVMs for robotic joint applications. This gap necessitates rigorous experimental benchmarking to establish performance tradeoffs and guide the development of next-generation actuators.

This paper presents a comprehensive comparative analysis of PMSM and PMVM topologies for high-performance robotic joint actuation systems. Section II establishes the design specifications and optimization framework for the proposed motors. Section III elucidates fundamental torque generation mechanisms by analyzing the effects of pole-slot combinations on torque performance, self-inductance, power factor, and efficiency. Section IV validates the theoretical analysis through experimental comparisons of back electromotive force (back-EMF), static torque, instantaneous torque, and efficiency. Section V concludes with critical insights.

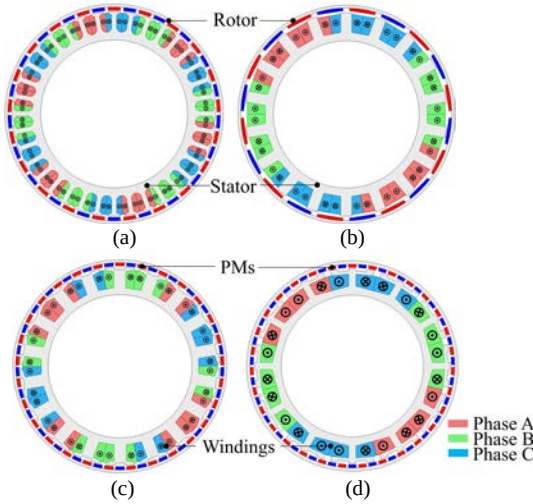


Fig. 1. Motor topologies. (a) 42p36s PMSM. (b) 20p18s PMVM. (c) 58p18s PMVM. (d) 62p12s PMVM.

II. DESIGN AND OPTIMIZATION OF ROBOT JOINT MOTOR

A. Topologies and Design Parameters

Robotic joint motors exhibit constrained overall dimensions governed by application-specific constraints. Multiple PMVMs with different stator split-tooth structures are designed and analyzed for comparison with a commercial 42-pole 36-slot (42p36s) PMSM. To ensure a fair evaluation, these designs maintain identical external dimensions and electrical loading specifications as the commercial 42p36s. Fig. 1 depicts representative topology configurations: (a)

commercial PMSM; (b)-(d) proposed PMVMs with non-split-tooth, double-split-tooth, and triple-split-tooth designs, respectively. Dimensional specifications for the stator and rotor components of PMSM and PMVMs are comparatively presented in Fig. 2. The parameters of the PMSM and proposed triple-split-tooth PMVM are summarized in Table I. The proposed joint motor features an external rotor configuration and an enlarged stator inner diameter. This design facilitates direct gearbox integration within the stator cavity, enabling compact joint structure. The robotic joint motor structure is illustrated in Fig. 3. The central shaft of the gearbox is integrated with the motor shaft. During rotor operation, the motor shaft drives the gearbox output shaft, delivering mechanical power externally. The following section provides a detailed description of the design and analysis process.

TABLE I
PARAMETERS OF PMSM AND PMVM

Parameters	Symbol	PMSM	PMVM
Rated speed/(r·min ⁻¹)	n_N	1500	
Rated torque/(N·m)	T_N	0.95	1.25
Rotor pole-pairs	p_r	21	31
Number of slots	Z_s	36	12
Gear ratio	k		6:1
Stator outer radius/mm	r_{os}	40.5	
Rotor outer radius/mm	r_{or}	43.5	
Length of air gap/mm	g	0.5	
Stack thickness/mm	l_{stk}	8.5	
Height of PMs/mm	w_{PM}	1.3	
Height of rotor yoke/mm	h_{yr}	1.2	
Number of series turns per phase	N_{ph}	192	
Winding connection	—	Δ	
Material of PMs	—	NdFeB-35	
Material of iron core	—	DW465-50	

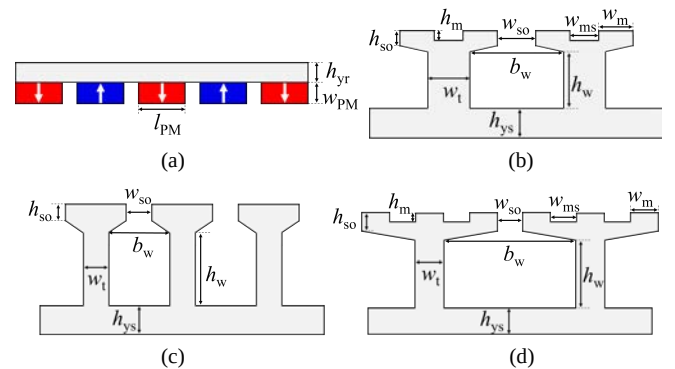


Fig. 2. Illustration of motor dimensions. (a) Rotor. (b) Stator of double-split-tooth. (c) Stator of PMSM. (d) Stator of triple-split-tooth.

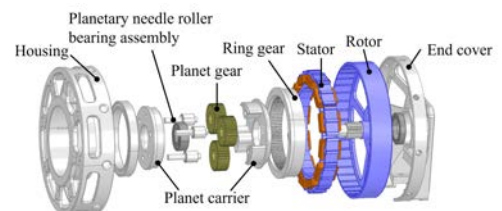


Fig. 3. Robotic joint motor structure explosion diagram.

B. Selection of the Pole-slot Combinations

The pole-slot selection for the PMVMs adheres to four principles: 1) maximized winding factor k_w ; 2) implementation of split-tooth topology; 3) high pole ratio G_r ; 4) selection of an appropriate number of slots. Non-split-tooth PMVMs employ an open-slot stator structure. When configured with an outer rotor (vs. an inner rotor), this topology increases the equivalent airgap length, reducing effective magnetic loading. Consequently, non-split-tooth PMVMs demonstrate better compatibility with inner-rotor architectures and ISDWs. Conversely, split-tooth topologies with narrower slot width demonstrate superior compatibility with outer-rotor architectures. FSCWs provide distinct advantages for joint motors requiring compact axial length through shortened end-windings and simplified coil distribution when benchmarked against ISDWs. Split-tooth PMVMs exhibit superior magnetic modulation efficacy vs. non-split-tooth topologies. Under equivalent volumetric constraints, split-tooth configurations achieve higher torque density. Therefore, this study adopts FSCW and split-tooth configurations. The short pitch winding factor k_p for the PMVMs is given by (1):

$$k_p = \sin\left(\frac{y_1 \pi}{\tau 2}\right) \quad (1)$$

where y_1 is the winding pitch and τ denotes the pole pitch defined as $\tau = Z_s/(2p_r)$. To maximize the winding factor, the short pitch factor should satisfy the condition specified in (2):

$$\frac{y_1}{\tau} = \frac{2p_r}{Z_s} = \frac{2(mZ_s - p_s)}{Z_s} = 2k + 1, \quad k = 0, 1, 2, \dots \quad (2)$$

where m is the number of split-tooth and p_s is the number of stator pole-pairs.

For PMVMs with non-split-tooth, double-split-tooth, and

triple-split-tooth configurations, the pole-slot combinations with higher winding factor are listed in Table II. Here, k_w denotes the winding factor, Z_f represents the number of modulating teeth, and G_r indicates the pole ratio, which can be expressed by (3):

$$G_r = \frac{p_r}{p_s} \quad (3)$$

$$E_{ph} = \frac{k_{dp1} N_{ph} D_g I_{ph} \omega_r}{\sqrt{2}} \left(B_{r0} + \frac{p_r}{p_s} B_{r1} \right) \quad (4)$$

where E_{ph} is the RMS value of EMF, k_{dp1} is the fundamental winding factor, D_g is the air-gap diameter, I_{ph} is the RMS value of the phase current, ω_r is the angular speed of the rotor, B_{r0} corresponds to the fundamental air-gap flux density of PMSM, and B_{r1} is the modulated air-gap flux density.

For PMVMs ($p_r \neq p_s$), (4) demonstrates theoretical torque proportionality to pole ratio G_r , though practical implementation constraints impose limits. Increasing p_r will raise the motor operating frequency, resulting in larger controller capacity and size, and increasing system costs. Therefore, the selection of pole ratio must strike a balance between torque enhancement and cost feasibility. The study constrains pole-pair counts to $p_r < 35$. To ensure comparative validity, all pole-slot configurations maintain constant electrical loading during design and optimization, enforcing identical turn counts per phase. Consequently, the number of slots is bounded by dual constraints. Excessive slots increase turn counts on each tooth, extending axial length, which affects robotic joint envelope dimensions. The ten pole-slot combinations meeting the criteria above are prominently displayed in blue in Table II.

TABLE II
MAIN POLE-SLOT COMBINATIONS

Non-split-tooth stator ($m = 1$)					Double-split-tooth stator ($m = 2$)								Triple-split-tooth stator ($m = 3$)									
p_r	10	8	11	10	17	13	22	23	14	28	19	29	34	14	21	29	22	38	23	31	24	32
Z_s	12	9	12	18	12	9	15	15	9	18	12	18	21	6	9	12	9	15	9	12	9	12
p_s	2	1	1	8	7	5	8	7	4	8	5	7	8	4	6	7	5	7	4	5	3	4
Z_f	12	9	12	18	24	18	30	30	18	36	24	36	42	18	27	36	27	45	27	36	27	36
G_r	5	8	11	1.25	2.43	2.6	2.75	3.29	3.5	3.5	3.8	4.14	4.25	3.5	3.5	4.14	4.4	5.43	5.75	6.2	8	8
k_w	1	0.95	0.93	0.95	0.93	0.95	0.95	0.95	0.95	0.95	0.93	0.90	0.89	0.87	0.87	0.93	0.95	0.95	0.95	0.93	0.87	0.87
y_1	3	4	5	1					1													

C. Optimization of the Joint Motor

Based on the pole-slot combinations in Table II and the impact of stator tooth geometry on magnetic modulation, the PMVM is optimized using particle swarm algorithms. Within the fixed external dimensions constraint, the dimensions of split-tooth—the parameter most critically influencing the motor's output torque capability are optimized. The selected design variables include stator tooth width w_t , modulated tooth height h_m , modulated tooth width w_m , stator yoke thickness h_{ys} , and slot opening width w_{so} , with dual optimization objectives of maximizing average torque T_{avg} and minimizing torque ripple T_{rip} . Stator tooth geometry must satisfy mechanical robustness requirements while maximizing electromagnetic performance. Within prescribed dimensional

boundaries, optimization variable ranges are initially defined via empirical design and preliminary finite element analysis (FEA). The parameter space exploration ranges for ten candidate motors are presented in Table III.

The comprehensive design framework for this joint motor optimization is illustrated in Fig. 4, incorporating two critical stages: 1) geometric parameter refinement for each pole-slot configuration through constrained optimization, and 2) performance-driven selection of the optimal electromagnetic topology. The comparative analysis of ten pole-slot combinations in Table IV reveals that the 62-pole 12-slot (62p12s) triple-split-tooth PMVM achieves superior torque characteristics, demonstrating a peak average torque of 1.25 N·m accompanied by a remarkably low torque ripple. Fig. 5

details the parametric sensitivity analysis through a gradient color scheme, where warmer hues indicate stronger correlations. This analysis establishes that the slot opening width w_{so}

exhibits the dominant influence on average torque, while the modulated tooth width w_m shows the highest correlation with torque ripple.

TABLE III
RANGE OF PARAMETERS

Motor	Range of parameters/mm				
	w_t	h_{ys}	w_{so}	w_m	h_m
34p12s	1.8–3.8	1.25–3.5	0.8–3	4–8	0.2–1.7
44p15s	2–5	0.75–2.75	0.5–3.5	1.2–6	0.5–2
46p15s	2–5	0.75–2.75	0.5–3.5	1.2–6	0.5–2
56p18s	2–5	1–3	1–4	2–5	0.5–2.7
38p12s	1.8–3.8	1.25–3.5	0.8–3	4–8	0.2–1.7
58p18s	2–5	1–3	1–4	2–5	0.5–2.7
68p21s	1–3.4	1.5–3.5	0.2–2.6	1.3–4.8	0.6–1.6
58p12s	1.8–3.8	0.65–2.85	0.8–4.8	1–5	0.45–1.45
62p12s	1.8–3.8	0.65–2.85	0.8–4.8	1–5	0.45–1.45
64p12s	1.8–3.8	0.65–2.85	0.8–4.8	1–5	0.45–1.45

TABLE IV
PARAMETERS AFTER OPTIMIZATION

Motor	Parameter/mm					Optimize objective	
	w_t	h_{ys}	w_{so}	w_m	h_m	$T_{avg}/(N \cdot m)$	T_{rip}
34p12s	3.8	3.25	2.8	7.7	0.47	0.86	3.58%
44p15s	5	2.75	1.5	4.23	0.65	1.07	2.92%
46p15s	5	2.75	1.5	4.73	0.89	1.13	2.25%
56p18s	5	2.75	2	4.47	0.7	1.08	3.74%
38p12s	3.8	3.25	2.8	6.2	0.69	0.94	2.60%
58p18s	5	2.75	2	3.97	0.88	1.09	1.02%
68p21s	3.4	3.3	1.4	3	0.98	1.18	1.77%
58p12s	3.8	2.45	1.6	2.54	0.83	1.23	1.70%
62p12s	3.8	1.92	1.82	2.53	0.87	1.25	1.01%
64p12s	3.8	2.45	1.6	3	0.86	1.16	7.22%

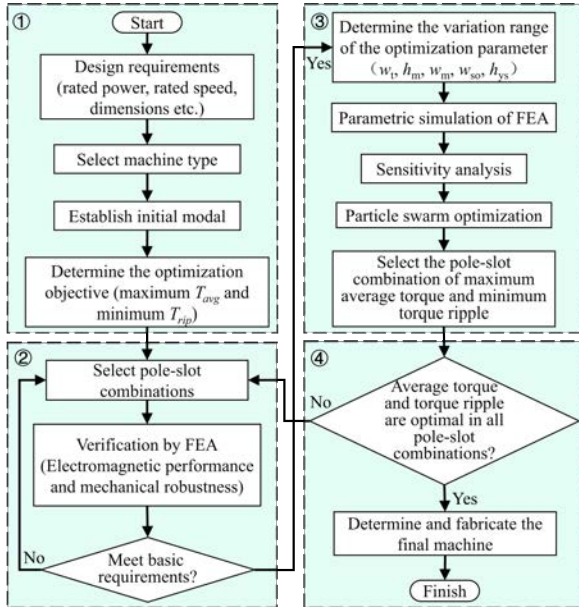


Fig. 4. Design and optimization process.

D. Influence of Parameters on Average Torque

Fig. 6 illustrates the parametric sensitivity trends of average torque with respect to stator tooth width w_t , modulated tooth height h_m , modulated tooth width w_m , and slot opening width w_{so} for four optimized motor configurations: 62p12s, 58p12s, 58p18s, and 68p21s.

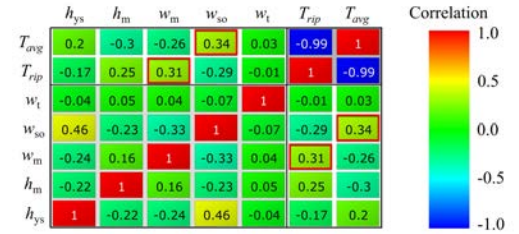


Fig. 5. Sensitivity analysis.

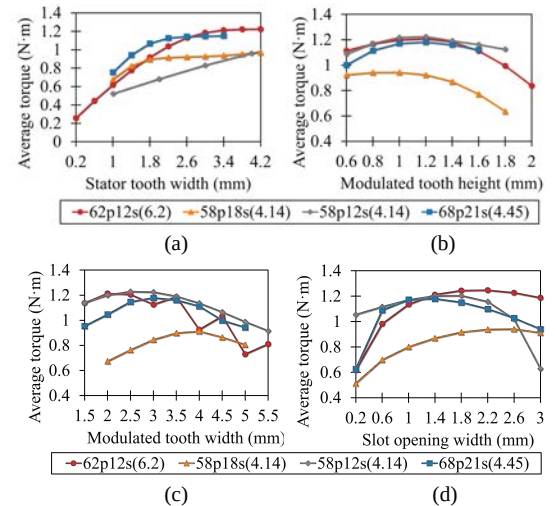


Fig. 6. Influence of parameters on average torque. (a) Stator tooth width. (b) Modulated tooth height. (c) Modulated tooth width. (d) Slot opening width.

1) Stator tooth width w_t : As shown in Fig. 6(a), the average torque demonstrates a characteristic saturation trend with increasing w_t across all motor configurations. The initial torque enhancement stems from improved magnetic circuit permeability and reduced flux saturation in stator teeth with width enlargement. However, magnetic saturation plateauing occurs beyond critical w_t values, resulting in diminishing torque gains.

2) Modulated tooth height h_m : Fig. 6(b) reveals a non-monotonic relationship between h_m and torque performance. The ascending phase correlates with enhanced air-gap flux modulation effectiveness through harmonic content optimization. Subsequent torque degradation at elevated h_m levels originates from dual detrimental effects: magnetic saturation in modulation teeth and aggravated permanent magnet flux leakage, both contributing to fundamental flux density attenuation.

3) Modulated tooth width w_m : The torque- w_m characteristic in Fig. 6(c) exhibits similar convex behavior. Optimal w_m balances modulation tooth desaturation and permanent magnet flux leakage mitigation. Excessive w_m expansion compromises flux modulation efficiency due to diminished magnetic reluctance contrast in the modulation structure.

4) Slot opening width w_{so} : As depicted in Fig. 6(d), the torque- w_{so} dependency reflects competing electromagnetic mechanisms. Moderate w_{so} enlargement enhances torque through harmonic content tailoring, whereas excessive openings induce parasitic slot leakage flux dominance and fundamental flux component deterioration. Notably, w_{so} intrinsically determines modulation tooth pitch under fixed geometric constraints.

III. EFFECT OF POLE-SLOT COMBINATION ON JOINT MOTOR PERFORMANCES

A. Principle

For PMVMs, the relationship of p_r and p_s satisfies [16]:

$$Z_f = mZ_s = p_r \pm p_s \quad (5)$$

where m is the number of split teeth.

As noted in [16], PMVMs exhibit higher torque capability when $Z_f = p_r + p_s$. All pole-slot combinations analyzed in this study satisfy this relationship. PMVMs achieve higher torque density than commercial PMSMs by satisfying (5) and leveraging harmonic magnetic fields—which commercial PMSMs cannot utilize for torque production. Based on (4), PMVM electromagnetic torque is derived as:

$$T_e = \frac{3k_w N_{ph} D_g I_{stk} I_{ph}}{\sqrt{2}} (B_{r0} + G_r B_{r1}) \quad (6)$$

Equation (6) shows the term $G_r B_{r1}$ represents an additional torque component generated by harmonic magnetic fields—absent in commercial PMSMs. Increasing this term directly enhances output torque. Substituting (5) into (3) yields:

$$G_r = \frac{mZ_s - p_s}{p_s} = m \frac{Z_s}{p_s} - 1 \quad (7)$$

Equation (7) demonstrates that with fixed Z_s and p_r ,

increased split-tooth number m elevates pole ratio G_r , thereby enhancing torque density. The essence of the field modulation effect of the PMVMs lies in the modulation effect on the magnetomotive forces (MMFs) by the air-gap permeance. For the split-tooth PMVM, the stator permeance is calculated by (8):

$$\lambda(\theta) = \lambda_0 + \sum_{l=1,2,3,\dots} \lambda_l \cos(lZ_s \theta) \quad (8)$$

where λ_0 is the DC component of the permeance, λ_l is the l th harmonic component of the permeance, and θ is the mechanical angle.

For PMSMs and non-split-tooth PMVMs, $Z_f = Z_s$. The critical distinction lies in the operational harmonic spectrum produced by their differing stator topologies after magnetic modulation. In non-split-tooth structures, the stator teeth simultaneously conduct flux and perform modulation, increasing susceptibility to magnetic saturation and reducing effective modulation ability. In contrast, split-tooth structures employ dedicated modulating teeth, achieving superior modulation performance and higher torque density. Then the no-load air-gap flux density b_r can be expressed as (9):

$$\begin{aligned} b_r(\theta, t) &= f_r(\theta, t) \cdot \lambda(\theta_s) = b_{\text{slotless}} \times \frac{\lambda(\theta)}{\lambda_{\text{slotless}}} \\ &= \sum_{v=1,3,5,\dots} F_{rv} \cos(v p_r \theta - v \omega_1 t) \cdot \left(\lambda_0 + \sum_{l=1,2,3,\dots} \lambda_l \cos(l Z_s \theta) \right) \\ &= \sum_{v=1,3,5,\dots} B_{rv0} \cos(v p_r \theta - v \omega_1 t) + \\ &\quad \sum_{v=1,2,3,\dots} \sum_{l=1,2,3,\dots} B_{rvl} \cos((v p_r \pm l Z_s) \theta - v \omega_1 t) \end{aligned} \quad (9)$$

where f_r is the MMFs generated by permanent magnets, F_{rv} is the v th amplitude of the f_r , ω_1 is the electric angular frequency, b_{slotless} is the slotless no-load air-gap flux density, and $\lambda_{\text{slotless}}$ is the slotless air-gap flux permeance. Equation (9) shows that the operational harmonics of the PMVM are p_r and $p_r \pm l Z_s$, whereas commercial PMSMs contain only the fundamental component p_r .

The electromagnetic torque generation mechanisms are analyzed below, utilizing the proposed 62p12s motor and the commercial 42p36s PMSM as examples. Fig. 7 compares the air-gap magnetic permeance waveforms along with their corresponding fast Fourier transform (FFT) spectra for both motors. It is observed that the proposed motor exhibits a dominant permeance harmonic at the $12l$ order, whereas the commercial baseline is characterized by a $36l$ -order harmonic. Crucially, the proposed motor possesses a richer spectrum of lower-order magnetic permeance harmonics. These components modulate the MMF into multiple torque-producing working harmonic fields that rotate at different speeds while sharing an identical electrical frequency. Furthermore, Figs. 8 and 9 illustrate the no-load radial air-gap magnetic flux density distributions derived via analytical derivations and FEA, respectively. For the commercial motor, only the 21st-order fundamental magnetic field contributes to the average torque, while the remaining harmonics act as parasitic components that induce torque ripple. In contrast, within the proposed motor, the air-gap magnetic field is modulated such that all labeled working harmonics can

participate in average torque production, where the 31st-order serves as the primary fundamental field. Notably, although the 5th-order harmonic acts as a detrimental source of torque ripple in the conventional design, its modulated counterpart in the proposed motor rotates at an amplified velocity of G_r times the rotor speed. This speed amplification enables the 5th harmonic to deliver a substantial average torque contribution without generating torque ripple. This mechanism elucidates why PMVMs inherently exhibit superior torque quality and lower ripples compared to conventional PMSMs.

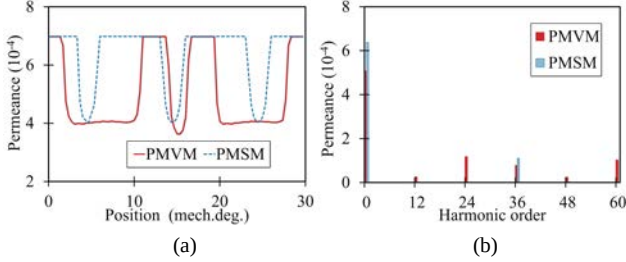


Fig. 7. Stator permeances of PMVM and PMSM. (a) Waveform. (b) Harmonics.

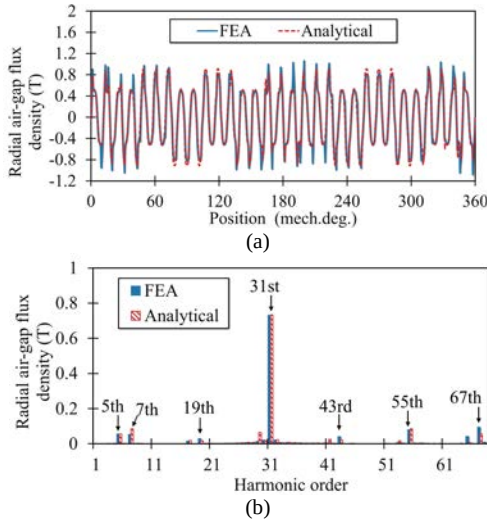


Fig. 8. FEA and analytical results of radial air-gap flux density under no-load of PMVM. (a) Waveforms. (b) Harmonics.

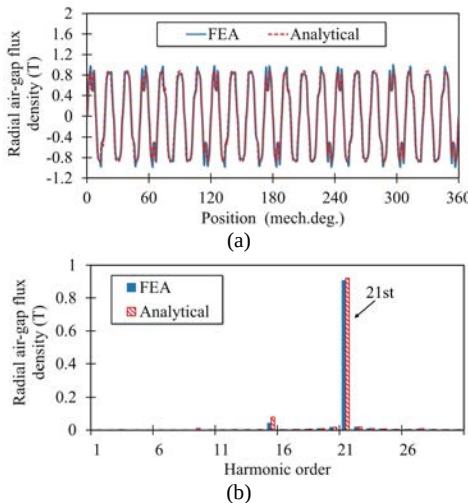


Fig. 9. FEA and analytical results of radial air-gap flux density under no-load of commercial PMSM. (a) Waveforms. (b) Harmonics.

B. Torque

Fig. 10 presents a comparative FEA investigation of instantaneous torque waveforms for double-split and triple-split tooth configurations across various pole-slot combinations, while Fig. 11 demonstrates the consistency between analytical predictions and numerical simulations regarding average torque characteristics vs. pole ratios. As evidenced by the results, both double-split and triple-split PMVM topologies exhibit a characteristic convex torque-pole ratio relationship under identical geometric and electrical loading conditions. This nonlinear behavior stems from competing electromagnetic mechanisms: 1) progressive magnetic leakage intensification between adjacent permanent magnets with rotor pole multiplication, and 2) a critical transition where the effective magnetic flux density degradation rate surpasses pole ratio growth. The latter phenomenon dominates beyond optimal pole ratios, triggering torque reduction through diminished air-gap flux utilization efficiency. Detailed comparative analysis reveals the following: 1) 34p12s vs. 38p12s: The 38p12s configuration achieves 25% torque enhancement at pole ratio 3.8 vs. 2.43, demonstrating beneficial pole scaling effects within subcritical ranges. 2) 46p15s vs. 56p18s: Despite a higher pole ratio (3.29 vs. 3.5), the 56p18s design suffers 4.76% torque reduction due to accelerated flux leakage from excessive rotor pole count.

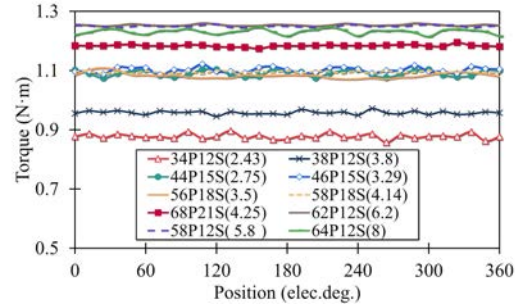


Fig. 10. Instantaneous torque of ten pole-slot combinations by FEA.

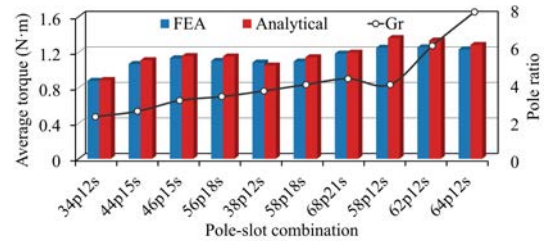


Fig. 11. Average torque of ten pole-slot combinations by FEA and analytical method.

C. Self-inductance

The equivalent magnetic circuits (EMC) of the double-split tooth and triple-split tooth PMVMs are established and shown in Fig. 12, neglecting the flux saturation and assuming the core permeability is infinite. When $p_s = 1$, the winding self-inductance L_s can be expressed as (10):

$$L_s = \frac{\psi_{AA}}{I_s} = \frac{N_s \Phi}{I_s} = \frac{N_s^2}{R_m} \quad (10)$$

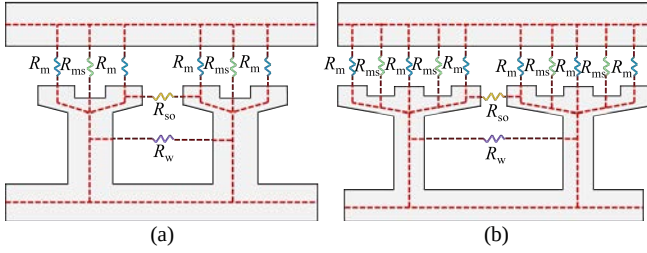


Fig. 12. EMC analytical models. (a) Double-split-tooth motor. (b) Triple-split-tooth motor.

where ψ_{AA} is the flux linkage of phase A, I_s is the winding current, N_s is the number of winding turns, ϕ is the flux, and R_m is the flux reluctance. The flux consists of the main flux ϕ_1 and the in-slot leakage flux ϕ_σ can be expressed as (11):

$$\Phi = \Phi_1 + \Phi_\sigma = \frac{2F}{3R_g} + \frac{2F}{R_\sigma} \quad (11)$$

where F is the MMF, R_g is the air-gap reluctance, and R_σ is the leakage reluctance. R_m , R_g , and R_σ are given by (12), (13), and (14), respectively.

$$R_m = \frac{F}{\Phi} = \frac{3R_g R_\sigma}{2(3R_g + R_\sigma)} \quad (12)$$

$$R_g = \begin{cases} \frac{R_{ms} R_m}{2R_{ms} + R_m}, & m = 2 \\ \frac{R_{ms} R_m}{3R_{ms} + 2R_m}, & m = 3 \end{cases} \quad (13)$$

$$R_\sigma = R_{s0} \parallel R_w \quad (14)$$

R_m , R_{ms} , R_{s0} , and R_w can be given by (15):

$$\begin{cases} R_m = \frac{g + w_{PM}}{\mu_0 w_m l_{stk}}, & R_{ms} = \frac{g + w_{PM} + h_m}{\mu_0 w_{ms} l_{stk}} \\ R_{s0} = \frac{w_{s0}}{\mu_0 h_{s0} l_{stk}}, & R_w = \frac{b_w}{\mu_0 h_w l_{stk}} \end{cases} \quad (15)$$

where μ_0 is the magnetic permeability of free space.

When $p_s \neq 1$, the self-inductance can be expressed as (16):

$$L_s = p_s \times \frac{N_s^2}{R_m} \quad (16)$$

Bringing (12)-(15) into (16), the winding self-inductance can be calculated, and the results compared with the FEA are shown in Fig. 13. It can be seen that the self-inductances of the triple-split-tooth motors are larger than those of the double-split-tooth motors and decrease with increasing pole ratio. Because of the following two points: 1) different permeances under different motor structures; 2) different p_s lead to different flux circuits, with large p_s , short flux circuits, and small self-inductance, while with small p_s , long flux circuits, and large self-inductance. Errors between FEA and analytical results in Fig. 13 are primarily due to neglected magnetic saturation in the analytical model. Notably, self-inductance trends from both methods match across pole ratios. Higher errors occur in high-saturation pole-slot combinations. For the four pole-slot combinations—34p12s, 38p12s, 64p12s, and 56p18s—the FEA results indicate that the degree of core saturation decreases progressively in the specified order.

Crucially, the discrepancy between the FEA and analytical results exhibits a consistent diminishing trend, demonstrating that the error of the analytical model primarily stems from magnetic saturation.

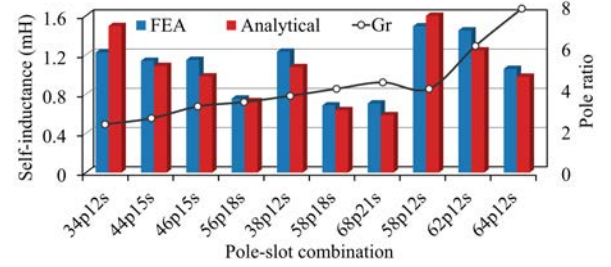


Fig. 13. Self-inductance of ten pole-slot combinations by FEA and analytical method.

D. Power Factor

Neglecting the winding resistance, the power factor of PMVM [22] aligns with that of commercial PMSM under $I_d = 0$ control and can be calculated by (17):

$$\cos \theta = \frac{1}{\sqrt{1 + (\omega_1 L_s I_s / E_0)^2}} = \frac{1}{\sqrt{1 + (L_s I_s / \psi_m)^2}} \quad (17)$$

where ω_1 denotes the electrical angular frequency, E_0 represents the no-load EMF, and ψ_m corresponds to the permanent magnet flux amplitude.

Equation (17) reveals that the power factor depends on L_s and ψ_m under constant stator current I_s . Specifically, a higher ψ_m or a lower L_s enhances the power factor. Fig. 14 presents the power factor and efficiency for ten pole-slot configurations calculated via FEA. The results indicate that the PMVM exhibits a relatively low power factor and efficiency. Moreover, motors with a double-split-tooth structure generally demonstrate higher power factor and efficiency than those with a triple-split-tooth structure. When the stator tooth count and stator pole pairs are identical, a higher pole ratio yields greater output torque but deteriorates the motor's power factor and efficiency. This conclusion is clearly demonstrated by comparing 34p12s vs. 58p12s ($Z_s = 12$, $p_s = 7$) and 38p12s vs. 62p12s ($Z_s = 12$, $p_s = 5$). This phenomenon arises from two inherent characteristics of PMVMs: 1) Flux leakage in open-slot structures: The open-slot design inherently increases flux leakage, substantially reducing ψ_m , which is a primary contributor to the low power factor. 2) Elevated winding inductance: Compared to the commercial PMSM (the simulated L_s is 0.497 mH), PMVMs exhibit higher winding inductance L_s , further diminishing the power factor.

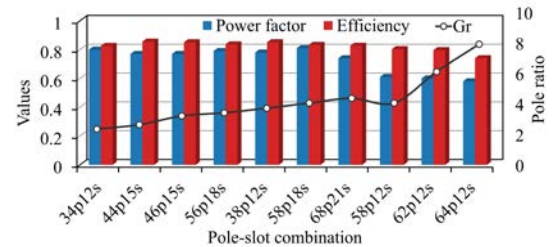


Fig. 14. Power factor and efficiency of ten pole-slot combinations by FEA.

IV. PERFORMANCE COMPARISON

A. Prototype and Test Bench

The prototype of the proposed 62p12s triple-split-tooth PMVM was fabricated for experimental benchmarking, based on design-phase analytical results and geometric parameters specified in Table I. A dedicated experimental platform was implemented to systematically evaluate the performance of both commercial PMSM and proposed PMVM topologies through comparative validation with FEA results. Both motor architectures underwent experimental characterization under identical instrumentation and standardized test protocols to ensure methodological consistency. The mechanical configurations of commercial and proposed joint motors are depicted in Figs. 15(a) and 15(b), respectively, while Fig. 15(c) presents the integrated experimental testbed configuration.

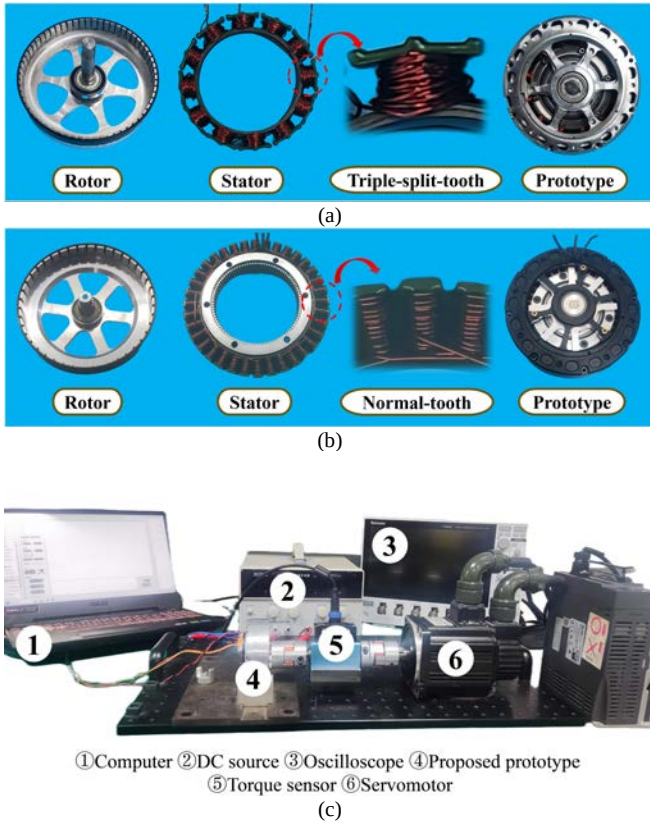


Fig. 15. Appearance of prototype and test bench. (a) Proposed prototype. (b) Commercial prototype. (c) Test bench.

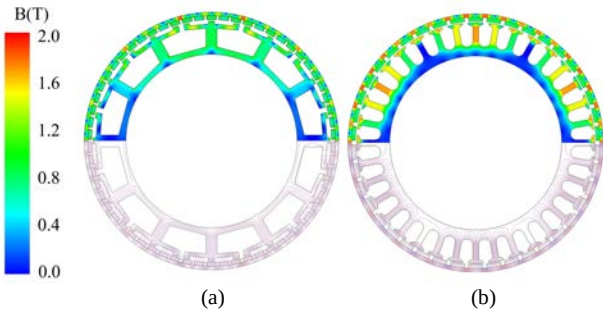


Fig. 16. Comparison of the no-load flux densities and flux lines. (a) Proposed joint motor. (b) Commercial joint motor.

The experimental test session comprises static and dynamic tests. The static test evaluates motor static torque using a servomotor, torque sensor, computer, and prototypes.

1) Static torque measurement: Single-phase winding receives direct current (DC) excitation. The servomotor rotates the prototype uniformly at low speed while measuring static torque at various rotor positions.

The dynamic test evaluates prototype no-load back-EMF, instantaneous torque, load current, output power, and efficiency using a servomotor, torque sensor, oscilloscope, controller, DC power supply, prototypes, and computer.

2) No-load back-EMF measurement: The servomotor drives the prototype to rated speed, and a precision oscilloscope captures the back-EMF.

3) Load performance measurement: The prototypes connect to a controller and DC power supply, and the servomotor applies specified load torque. The instantaneous torque, current, and voltage can be measured by a torque sensor and oscilloscope, respectively. Then the mechanical power and electrical power can be calculated, and efficiency can be determined.

B. Comparison of No-load Performances

The no-load flux density distributions and magnetic flux lines of both joint motors are presented in Fig. 16. While exhibiting equivalent rotor yoke saturation levels, the commercial joint motor demonstrates more pronounced magnetic saturation in its stator teeth compared to the proposed design. Furthermore, the proposed motor configuration exhibits extended flux path lengths within its stator yoke structure. The no-load three-phase line back-EMF waveforms of commercial and proposed joint motors are shown in Figs. 17 and 18. The measured waveforms exhibit a high degree of symmetry with near-sinusoidal characteristics. Fig. 18 demonstrates a substantial enhancement in the fundamental harmonic amplitude of the proposed motor's back-EMF compared to the commercial motor. Further harmonic analysis reveals distinct differences in harmonic distribution: In the proposed motor, the 3rd and 5th harmonic components account for 3.17% and 1.47% of the total harmonic content, respectively. This contrasts with the commercial motor's harmonic composition, where the 3rd and 5th harmonics constitute 0.44% and 2.02%, respectively. The total harmonic distortion (THD) values for the proposed motor and the commercial motor are 6.72% and 2.27%, respectively. The comparison of the measured and simulated no-load back-EMF amplitude of the two motors at different speeds is shown in Fig. 19, where the amplitude of the back-EMF of the proposed motor at the same speed is larger than that of the commercial motor. The results of the measured and simulated static torque of the two motors are shown in Figs. 20 and 21, respectively. Fig. 20 shows the static torque for applying 6 A current to the single-phase winding, and the proposed motor has lower ripple and higher amplitude than the conventional motor. The static torque amplitude of the two motors for different phase currents is shown in Fig. 21, which matches the simulation results.

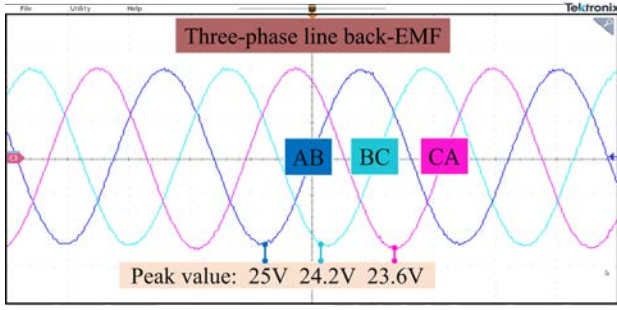


Fig. 17. Measured three-phase line back-EMF of the proposed joint motor at 1500 r/min under no-load condition.

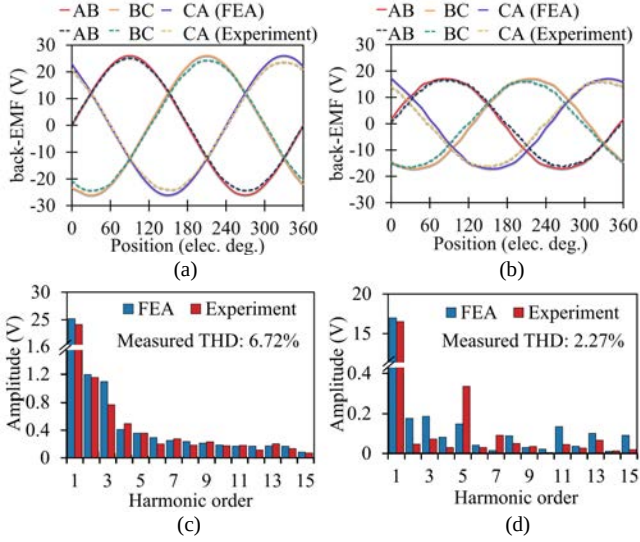


Fig. 18. Comparison of the three-phase line back-EMF at 1500 r/min under no-load condition. (a) Waveform of the proposed joint motor. (b) Waveform of the commercial joint motor. (c) Harmonic amplitude of the proposed joint motor. (d) Harmonic amplitude of the commercial joint motor.

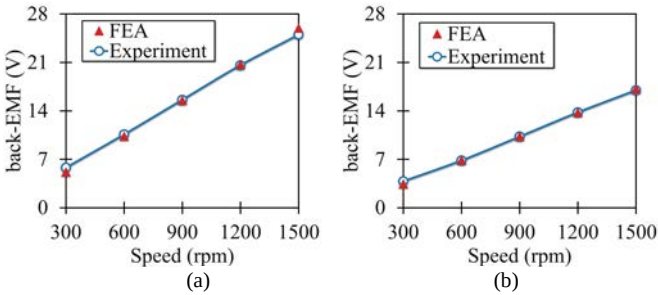


Fig. 19. Comparison of the three-phase line back-EMF at different speeds under no-load conditions. (a) Proposed joint motor. (b) Commercial joint motor.

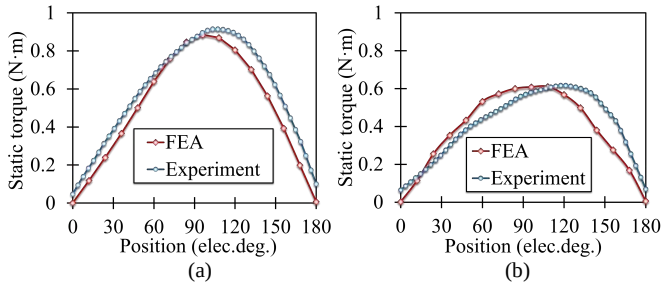


Fig. 20. Comparison of the static torque. (a) Proposed joint motor. (b) Commercial joint motor.

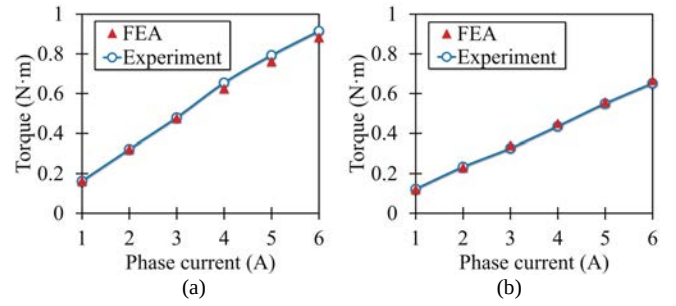


Fig. 21. Comparison of the peak values of static torque under different currents. (a) Proposed joint motor. (b) Commercial joint motor.

C. Comparison of Rated Load Performances

The comparative framework evaluating the relative merits of both motors through quantitative analysis of instantaneous torque, torque-current characteristics, efficiency, total active mass, and torque density is established. The results are summarized in Table V. Fig. 22 shows the measured three-phase line current of the proposed joint motor under the rated load of 1.25 N·m. The results of the measured and simulated instantaneous torque of the two motors under rated load conditions are shown in Fig. 23. It can be seen that the torque ripple of the proposed joint motor is lower than that of the commercial joint motor. The average output torque of the two motors at different RMS values of line current is shown in Fig. 24. The proposed joint motor has larger output torque at the same current, and the results of the simulation and measurement correspond. Commercial PMSM exhibits slightly higher efficiency than the PMVM. The slight reduction in the proposed motor's efficiency compared to the commercial PMSM baseline stems from higher core and copper dissipations. At rated speed, the 62-pole proposed motor experiences a high electrical frequency of 775 Hz while the 525 Hz baseline of the 42-pole counterpart, causing elevated frequency-dependent iron losses. Additionally, despite carrying an equivalent fundamental electrical load, the proposed motor yields a larger back-EMF THD (Fig. 18), introducing parasitic harmonic fields that induce supplementary winding losses. Structurally, the open-slot configuration combined with a high rotor pole count increases both slot and inter-pole leakage flux profiles. This behavior is accompanied by a higher self-inductance value, since the proposed machine operates with a small fundamental stator pole-pair count (5 vs. 21), leading to longer flux loop paths (Fig. 16). Consequently, the power factor of the proposed motor is lower than that of the commercial motor due to higher self-inductance. Importantly, achieving elevated torque density and superior torque quality is critical for joint motors to ensure rapid dynamic response and smooth kinematic trajectory tracking. The pole ratio (G_r) stands as the pivotal indicator governing the trade-offs among torque propagation, efficiency, and power factor alignment. While increasing G_r effectively enhances the output torque via magnetic field modulation, an excessive G_r accelerates the operational frequency, inflating core losses and deteriorating the power factor. This degradation expands the required drive inverter

capacity and premiumizes system costs. Therefore, systematically optimizing the selection of G_r during the preliminary design phase is critical to balancing the multi-attribute performance of the joint motor. The weights and torque densities of the two motors are shown in Fig. 25. The weight of the proposed motor is slightly heavier than that of the commercial motor, while the torque density is increased to 1.24 times.



Fig. 25. Comparison of the torque density.

TABLE V
MEASURED PERFORMANCE COMPARISON OF TWO MOTORS

Parameters	PMVM	PMSM
Average torque/(N·m)	1.25	0.95
Torque ripple/%	4.92	5.27
Efficiency/%	76.96	80.59
Total active mass/g	544.5	507.5
Torque density/(N·m·kg ⁻¹)	2.30	1.852

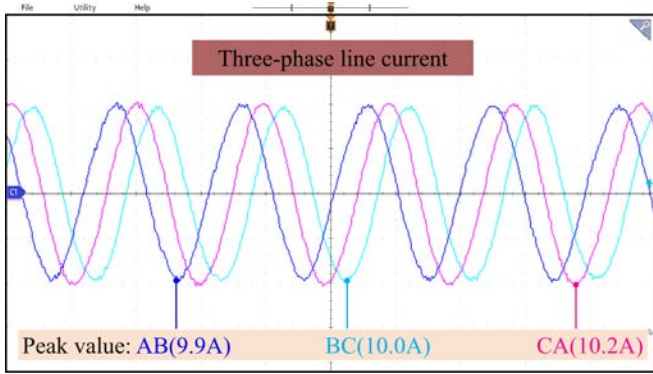


Fig. 22. Measured three-phase line current of proposed joint motor under rated load.

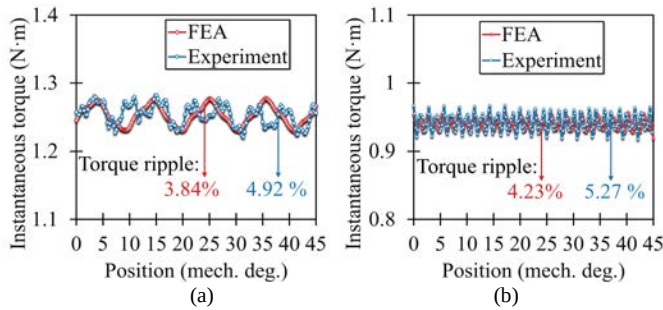


Fig. 23. Comparison of the Instantaneous torque under rated load. (a) Proposed joint motor. (b) Commercial joint motor.

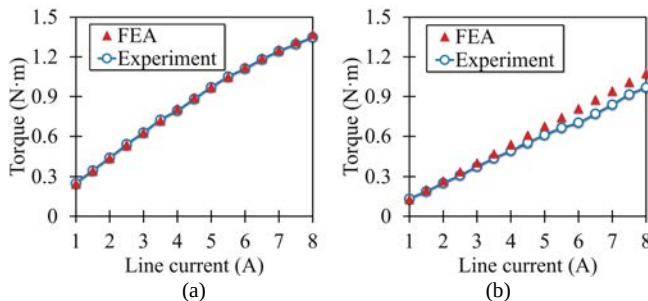


Fig. 24. Comparison of the average values of instantaneous torque under different currents. (a) Proposed joint motor. (b) Commercial joint motor.

V. CONCLUSION

With the purpose of enhancing the torque performance of robotic joint motors, this paper introduces the PMVM topologies. Various pole-slot configurations are investigated, and multi-objective optimization is performed. This ultimately leads to the design and fabrication of a 62p12s, triple-split-tooth PMVM. Following a comprehensive comparison with a commercial 42p36s PMSM—through theoretical derivations, FEA, and experimental testing—the following conclusions are drawn:

- 1) For the split-tooth PMVM, the slot opening width w_{so} exerts the dominant influence on average torque, whereas the modulated tooth width w_m exhibits the strongest correlation with torque ripple.
- 2) Within a specific range, increasing the pole ratio G_r effectively improves the torque density of the PMVM. However, this adjustment may adversely affect key parameters (power factor, efficiency) and thus should be comprehensively considered in the motor design process.
- 3) The proposed 62p12s triple-split-tooth PMVM achieves a 24.2% torque density improvement and a 6.6% lower torque ripple compared to a commercial PMSM.

The lower power factor results in lower efficiency and relatively higher system costs for PMVMs. The Halbach array with consequent-pole rotor topology may be a promising direction, and the work is now underway.

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Rui Li was born in Jiangxi, China. He received the B.Sc. degree in electrical engineering from Shenyang University of Technology, Shenyang, China, in 2024. He is currently working toward the M.Sc. degree in electrical engineering with the School of Electrical Engineering, Shandong University, Jinan, China.

His research interests include the design of high-performance joint motors.



Daohan Wang (Senior Member, IEEE) received the B.Eng., M.Eng., and Ph.D. degrees in electrical engineering from Shandong University, Jinan, China, in 2003, 2006, and 2010, respectively.

Since 2013, he has been with Shandong University, where he is currently a Professor with the School of Electrical Engineering. His research interests include application-oriented electrical drives and sustainable energy conversion.



Yingsa Li was born in Jinzhou, Liaoning, China, in 1996. He received the M.S. degree in electrical engineering from Shenyang University of Technology, Shenyang, China, in 2022. He is currently pursuing the Ph.D. degree in electrical engineering with Shandong University, Jinan, China. His research interests

include electromagnetic and vibration of permanent magnet (PM) motors.



Dandan Song was born in Weihai, Shandong, China, in 1998. She received the M.Eng. degree from Northeastern University, Shenyang, China, in 2024.

She is currently working toward the Ph.D. degree in electrical engineering with the School of Electrical Engineering, Shandong University, Jinan, China. Her current research interests include fault diagnosis of incipient faults in PMSM.



Ran Zhang received his graduation degree from Shandong University, Jinan, China, in 2010. He was with the Shandong University of Aeronautics, Jinan. He is currently a Doctor of Electrical Engineering and a Vice Professor. His research focuses on control and simulation of permanent magnet motors.



Xiuhe Wang (Member, IEEE) was born in China in July 1967. He received the B.E. and M.E. degrees from Shandong University, Jinan, China, in 1989 and 1993, respectively, and the Ph.D. degree from the Shenyang University of Technology, Shenyang, China, in 1996, all in electrical engineering. From 2001 to

2002, he was a Postdoctoral Fellow at Seoul National University, Seoul, South Korea.

He is currently a professor of electrical engineering and the Vice Head of the School of Electrical Engineering, Shandong University.